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Summer Convention

Five-Band Automobile Receiver

Electron Microscope

Stabilized Frequency Divider

Vibrations in Quartz Plates

Acoustic Models of Radio Antennas

Damped Vibrations of Quartz Plates

Large-Signal High-Frequency Electronics

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A Five-Band Receiver for Automobile Service*

J. H. LITTLE†, NONMEMBER, I.R.E., AND F. X. RETTENMEYER‡, MEMBER, I.R.E.

Summary—*Many sections of the United States are without adequate daylight radio program service from the standard broadcast stations. These areas in the main are located in the southern part of the country, where the static level is higher and continues for a longer portion of the year, in the Rocky Mountain states where radio stations are scarce, and in New England. Many of these areas contain resorts and represent the most popular recreational areas in the country. Since automobile receivers are mobile, this particular situation becomes serious for the touring public.*

At the present time there are some 30,000,000 registered automobiles in the United States of which 27 per cent are equipped with radio receivers. One possibility of reception in the areas not now adequately served with broadcast stations lies in the use of short waves in the international short-wave band. This paper describes a receiver intended for this type of service. It is believed that the development of this receiver has opened up markets not heretofore available in this field. It is suggested that an economical approach to coverage of the large areas not now served by broadcast stations might be accomplished through the employment of present domestic international short-wave stations with or without directional antennas for this purpose. These stations might be operated at frequencies not usable at the time for intercontinental broadcasting. For example, when the 16-meter band is in use for intercontinental broadcasting, the 19- or 25-meter band might be used for coverage of certain areas in the United States.

Broadcast Frequency Coverage of the United States

THERE ARE large areas of the continental United States which are without adequate daylight radio program service from standard broadcast stations. Indeed, there are appreciable areas without any daytime service at all and with, at best, doubtful nighttime service. In its Fourth Annual Report[1] the Federal Communications Commission estimated that in 1938 some 38 per cent of the area of the continental United States, which included some 8 per cent of the total population, was outside the daytime service areas of standard broadcast stations. This report also indicated that in 1938 some 56 per cent of the area of the United States including some 17 per cent of its population was outside the nighttime service areas of standard broadcast stations. While these figures may not represent conditions that exist today as accurately as they represented conditions existing in 1938, they are at least generally indicative of a situation that does exist now. In many localities there appears to be no immediate improvement of this situation in sight because of the economic considerations peculiar to these localities. There are many cities and towns as well as rural areas which do not have facilities of any sort. Evidence seems to indicate that, in general, stations located in towns having a population of less than 10,000 cannot expect to receive sufficient financial support to sustain a high quality of program service over an extended period of time, unless they are in the center of a distributed population having a much greater purchasing power than the town's population alone would indicate. The Fifth Annual Report[2] of the Federal Communications Commission for the year ending June 30, 1939, states that, "There are 597 towns having a population in excess of 10,000 which do not have radio stations. Of this number, 464 towns, each with a population between 10,000 and 25,000 and 133 with a population in excess of 25,000, are without radio stations."

This same report further shows that in 1939 there were some 380 cities and towns in the continental United States which had only one radio station. Of the more than 2000 cities in the United States having a population between 2500 and 10,000, it was estimated that 725 (with a total population of 3,487,000) do not have adequate signals from even one station during the daytime, and that 854 (having a total population of 4,138,000) do not have adequate signals from even one station during the nighttime. In addition, there are a considerable number of towns and portions of towns having a population of less than 2500 which do not receive adequate signals from even one station and could not be expected to support a station even though facilities were available.

The greatest need for improved signal intensity and improved coverage is in the southern regions of the country where the static level is the highest and where the high static level continues for a longer portion of the year than in the other parts of the country. In the Rocky Mountain states, where the population is scattered and radio stations are scarce, there is also a great need for improvement insofar as adequate daylight coverage is concerned. Fig. 1 shows the distribution of population in the United States. Fig. 2 shows[3] the daylight coverage for standard broadcast stations in the continental United States. Fig. 3 shows[3] the nighttime coverage for standard broadcast stations in the United States. It is obvious from these data that a large part of the continental United States, and particularly that portion which represents the recreational areas, is not now adequately served by standard broadcast stations.

Of course, the distribution of population has less significance for auto radio receivers than for home receivers. Moreover, auto radio tends to augment the large concentrations of the radio listening population in recreational areas during certain seasons of the year. The automobile receiver, being mobile, adds another serious aspect to this situation in that the receiver may

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[1] Fourth Annual Report of the Federal Communications Commission for the year ending June 30, 1938.

[2] Fifth Annual Report of the Federal Communications Commission for year ending June 30, 1939.

[3] Figs. 2 and 3 were taken from the Fourth Annual Report of the Federal Communications Commission.

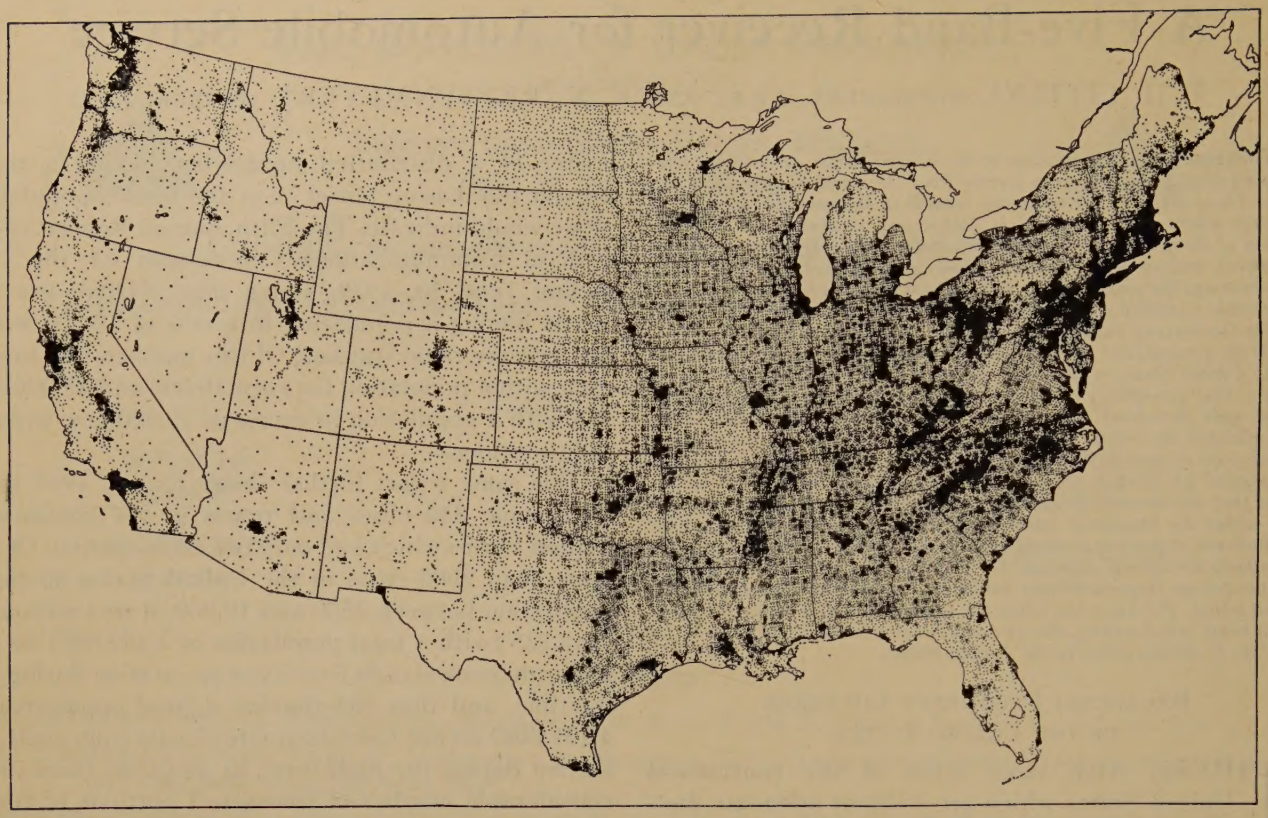

Fig. 1—Population distribution of the United States.

move through territories inadequately served by standard broadcast stations. At the present time, there are some 30,000,000 registered automobiles in the United States. Of these some 8,000,000 or about 27 per cent are equipped with radio receivers. This is indicative of the importance of automobile radio in the social life of our population.

From the foregoing, it is obvious that large masses of the population cannot be served properly either during the day or night with standard broadcast-band auto receivers. Moreover, reception of broadcast stations, even in areas of relatively strong signals may be extremely poor during electrical storms. Short-wave reception in the international short-wave broadcast bands presents one solution to this problem in that: (a) short waves are less affected by static than long waves; (b) short-wave reception is available both during daylight and darkness hours in a large portion

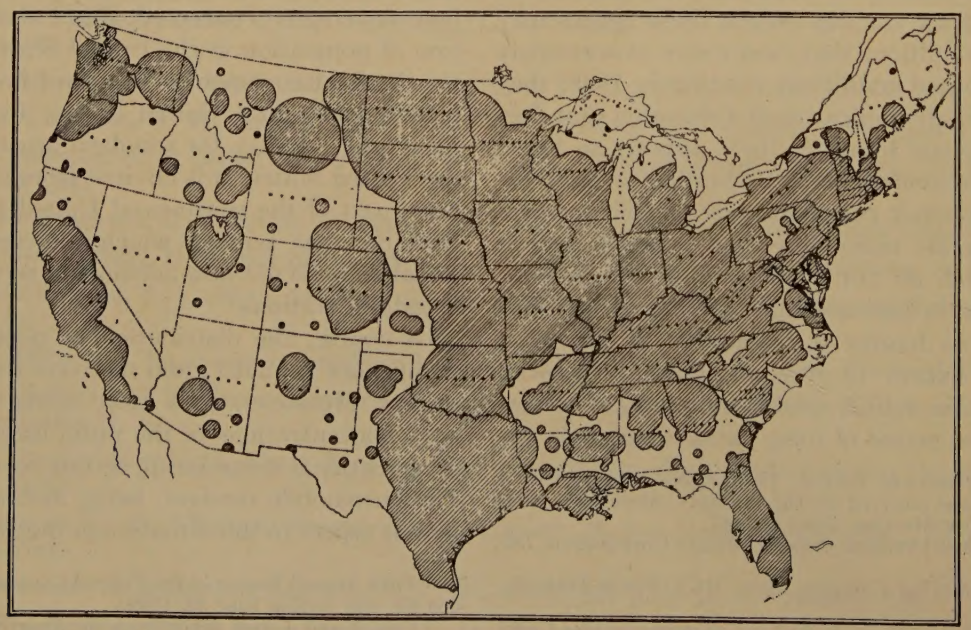

Fig. 2—Daytime service areas of all standard broadcast stations in the United States.

of the continental United States; (c) short waves are less affected by violent electrical storms than the stations in the broadcast band; and (d) short waves often provide a greater variety of programs than otherwise would be available.

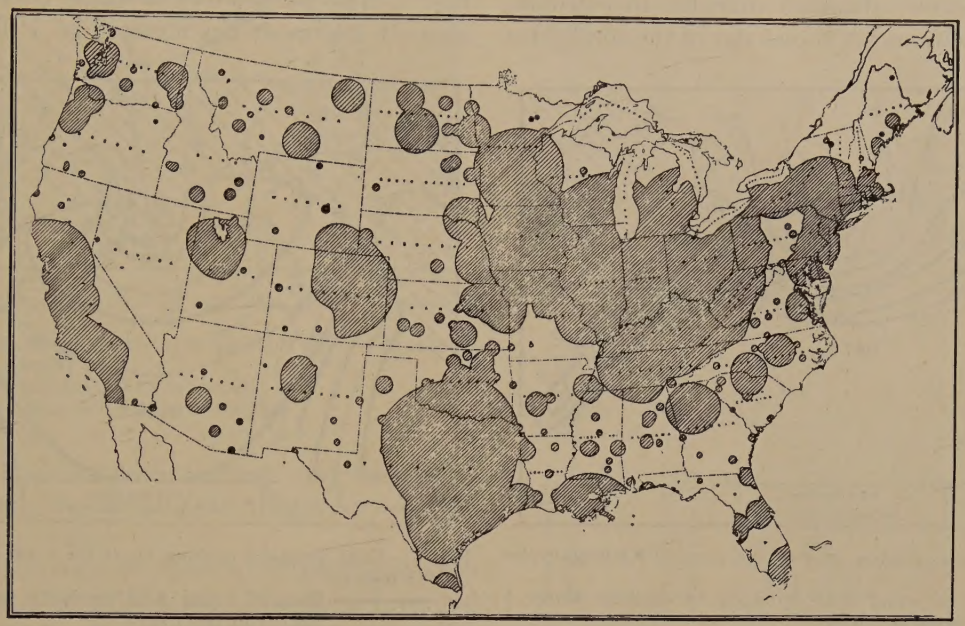

Fig. 3—Nighttime service areas of all standard broadcast stations in the United States.

Short-Wave-Propagation Phenomena

The chief limitation of short-wave reception is the variability of transmission. The factors affecting radio transmission are many and complex. Consequently, any simple generalization of the subject must necessarily be undertaken at the expense of thoroughness and accuracy of detail.

In considering radio transmission, we are concerned with two fundamental variables. The first is the electrical noises that occur as a result of atmospheric disturbances. The second is the attenuation of the signal with the spread of transmitted energy, its absorption by the earth and upper atmosphere, and its redistribution as affected by ground reflections. Consideration of static interference in this paper will be limited to the noises which arise from atmospheric sources, principally lightning discharges. The other general type of noise which arises largely from industrial sources, is so varied in characteristic and so much a matter of specific location and time that it does not readily lend itself to simple or brief treatment. In general, the intensity of atmospheric static[4] throughout the radio spectrum decreases with increase in frequency. This effect is illustrated[5] by Fig. 4. This particular figure relates to reception of static interference from a near-by thunder storm. Curve *A* is for reference purposes and is an inverse-frequency curve. Curve *B* for midnight conditions shows a trend with frequencies which is similar to that of curve *A* except at the higher frequencies where the ionization of the upper atmosphere limits transmission. Curve *C* for midday conditions shows a rise in static level in the 19-meter range. This rise is due to the fact that reflections from the ionosphere become prominent for these waves enabling them to propagate over great distances by so-called sky-wave transmission. The greater transmission range of these waves means that the static is collected from a greater area resulting in an increase in the noise level corresponding to the improved effectiveness of the waves in this position of the frequency spectrum. At

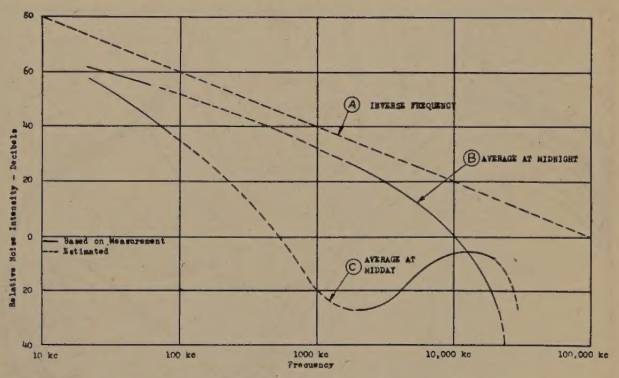

Fig. 4—Relative variation of atmospherics with frequency for northeastern United States.

frequencies higher than those shown in Fig. 4, the noise level from atmospheric sources continues to decrease.

Radio transmission at short waves is characterized by high ground-wave attenuation and small sky-wave attenuation. Due to the complicated nature of the variation in the ionosphere of layers which determine

[4] F. E. Lutkin, "The nature of atmospherics," *Proc. Royal Soc.* (London), vol. 171, p. 285; June, 1939.

[5] Fig. 4 is reprinted from "Radio transmission in relation to aeronautics," *Jour. Inst. Aer. Sci.*, March, 1935, courtesy of Lloyd Espenschied.

the transmission, the propagation variations are so complex as to preclude a comprehensive presentation of the facts of high-frequency transmission in any small number of charts. If ionospheric data were completely known over all the earth's surface at all seasons it might be possible, although difficult, to calculate propagation conditions.[6] A knowledge of the conditions of the ionosphere is most comprehensively expressed in terms of data which relate to the virtual height of the layer and to frequency. These data are usually of a relatively high order of accuracy for any particular time and place.

Fig. 5 is a field-strength contour map[7] of a 1-kilowatt London station operating into a half-wave antenna at a wavelength of 35 meters. Sunrise and sunset are indicated by the dotted curve. Sunrise occurs at *L* and sunset at *C*. The crosshatched areas are skip regions of low intensities. Reception in the skip regions is uncertain although it may be had due to reflections, etc. Fig. 6 illustrates another field-intensity contour map for 35 meters taken 6 hours after the curves of Fig. 5. The parameters of these curves are decibels above 1 microvolt per meter. Fig. 7 illustrates conditions during nighttime for 35 meters. Fig. 8 is a field-intensity chart for 16 meters from a 1-kilowatt London transmitter operating into a half-wave antenna. It is representative of winter morning conditions on the North American continent. Fig. 9 represents reception conditions on the North American continent at midday on 16 meters. Figs. 5 to 9, inclusive, relate to reception from England. They are some two years old and are reprinted here only since they are representative to some extent of existing conditions. The chart of Fig. 10 summarizes data from field-strength contour maps. It

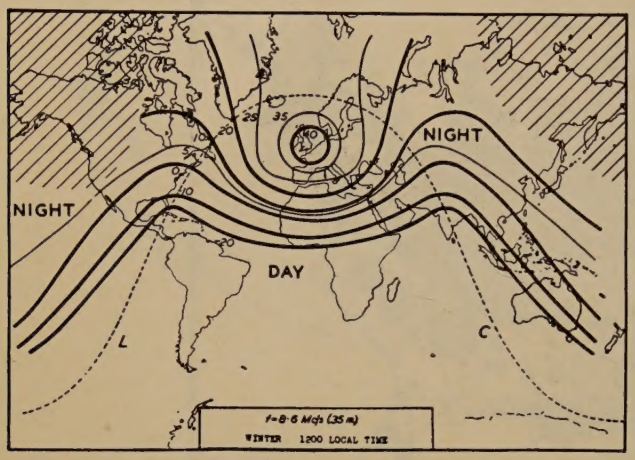

Fig. 5—Field-intensity contour chart for a wave of 8.6 megacycles (35 meters).
——— lines of equal field intensity in decibels above 1 microvolt per meter
– – – locus sunrise (L) to sunset (C)
/ / / / / skip region

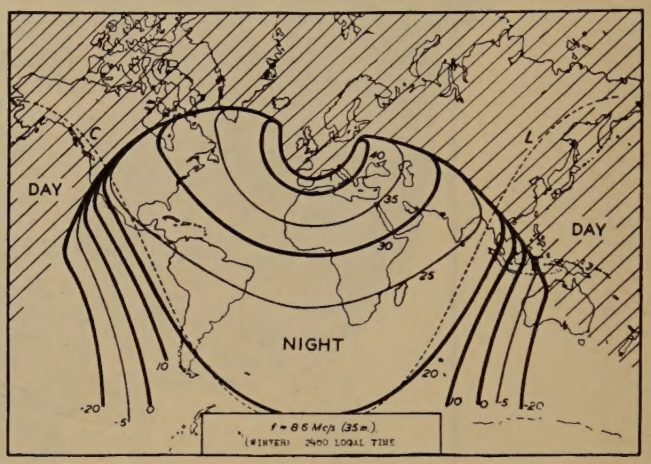

Fig. 7—Field-intensity contour chart for a wave of 8.6 megacycles (35 meters).
——— lines of equal field intensity in decibels above 1 microvolt per meter
– – – locus sunrise (L) to sunset (C)
/ / / / / skip region of zero field intensity

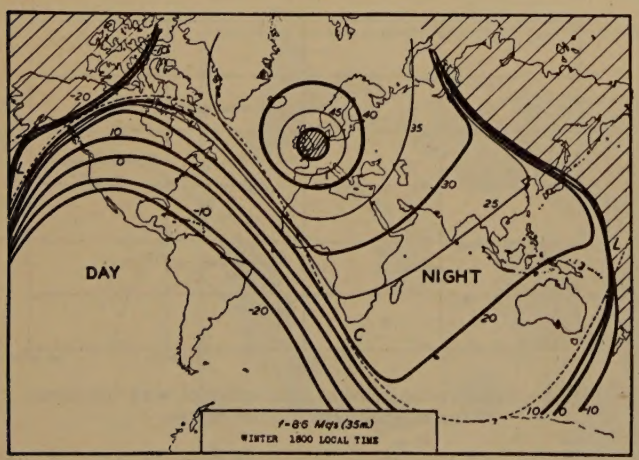

Fig. 6—Field-intensity contour chart for a wave of 8.6 megacycles (35 meters).
——— lines of equal field intensity in decibels above 1 microvolt per meter
– – – locus sunrise (L) to sunset (C)
/ / / / / skip region of zero field intensity.

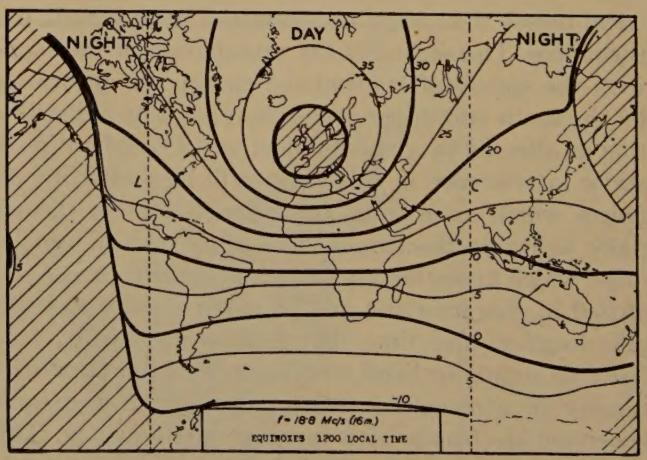

Fig. 8—Field-intensity contour chart for a wave of 18.8 megacycles (16 meters).
——— lines of equal intensity in decibels above 1 microvolt per meter
– – – locus sunrise (L) to sunset (C)
/ / / / / skip region

[6] Bureau of Standards Circular No. 317.
[7] Reprinted from "Report of committee on radio wave propagation," Proc. I.R.E., vol. 26, pp. 1193–1234; October, 1938.

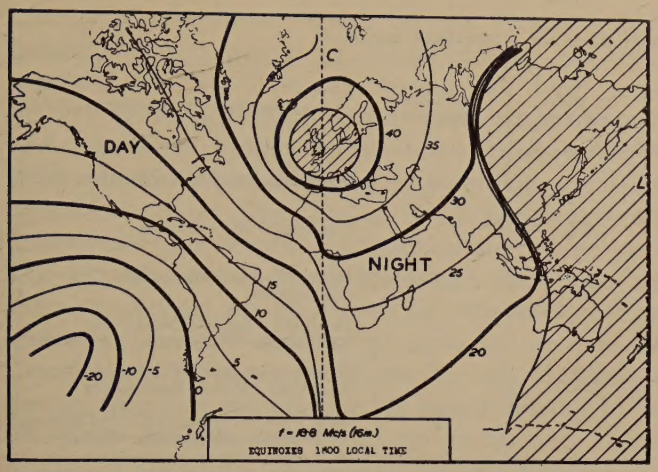

Fig. 9—Field-intensity contour chart for a wave of 18.8 megacycles (16 meters).
——— lines of equal intensity in decibels above 1 microvolt per meter
– – – locus sunrise (L) to sunset (C)
/ / / / / skip region of zero field intensity

is approximate and can serve, at best, only as a useful guide to determine the optimum frequency for a known distance and time of day.

A detailed discussion of short-wave propagation is beyond the scope of this paper. The data included herein are intended only to indicate the possibilities of short-wave reception insofar as it affects auto radio receivers. More detailed information may be found in some of the appropriate references mentioned in the bibliography.

TABLE I

TYPICAL SHORT-WAVE PROGRAM FOR EUROPEAN SHORT-WAVE STATION

Broadcast Periods			
6:00 A.M.— 9:00 A.M.	DJB	15.20 Mc	19.74 m
4:50 P.M.—11:00 P.M.	DJD	11.77 Mc	24.49 m
5:45 P.M.— 6:35 P.M.	DZD	10.54 Mc	28.45 m
7:00 P.M.—11:00 P.M.	DZD	10.54 Mc	28.45 m
8:00 P.M.— 1:00 A.M.	DJC	6.02 Mc	49.83 m

Standard News Periods

E.S.T.	P.S.T.		E.S.T.	P.S.T.	
6:30 A.M.	3:30 A.M.	English	5:30 P.M.	2:30 P.M.	English
8:00	5:00	German	5:45	2:45	German
8:30	5:30	English	6:00	3:00	English
11:30	8:30	English	8:00	5:00	German
12:30 P.M.	9:30	English	8:15	5:15	English
1:30	10:30	English	10:15	7:15	German
3:30	12:30 P.M.	English	10:30	7:30	English
3:45	12:45	French	11:15	8:15	Spanish
4:30	1:30	English	12:00	9:00	English

Sunday, December 22

E.S.T.	P.S.T.	
4:50 P.M.	1:50 P.M.	Call, German Folk Songs
4:55	1:55	Announcement of the Day's Program
5:00	2:00	"Nuernberger Rauschgold" (Nuremberg Tinsel) A Children's Program
5:30	2:30	"The Zeesen Women's Club"
6:15	3:15	Popular Opera Concert
7:15	4:15	News Review in French
8:30	5:30	"Lord Haw-Haw Speaks to England"
8:45	5:45	Sunday Evening Program
9:00	6:00	Rear-Admiral Lützow
9:20	6:20	"Action Reports from the Front"
9:30	6:30	"Anchors Aweigh"
10:50	7:50	Talk in English

Table I shows a typical short-wave program of a European station. It will be obvious from an inspection

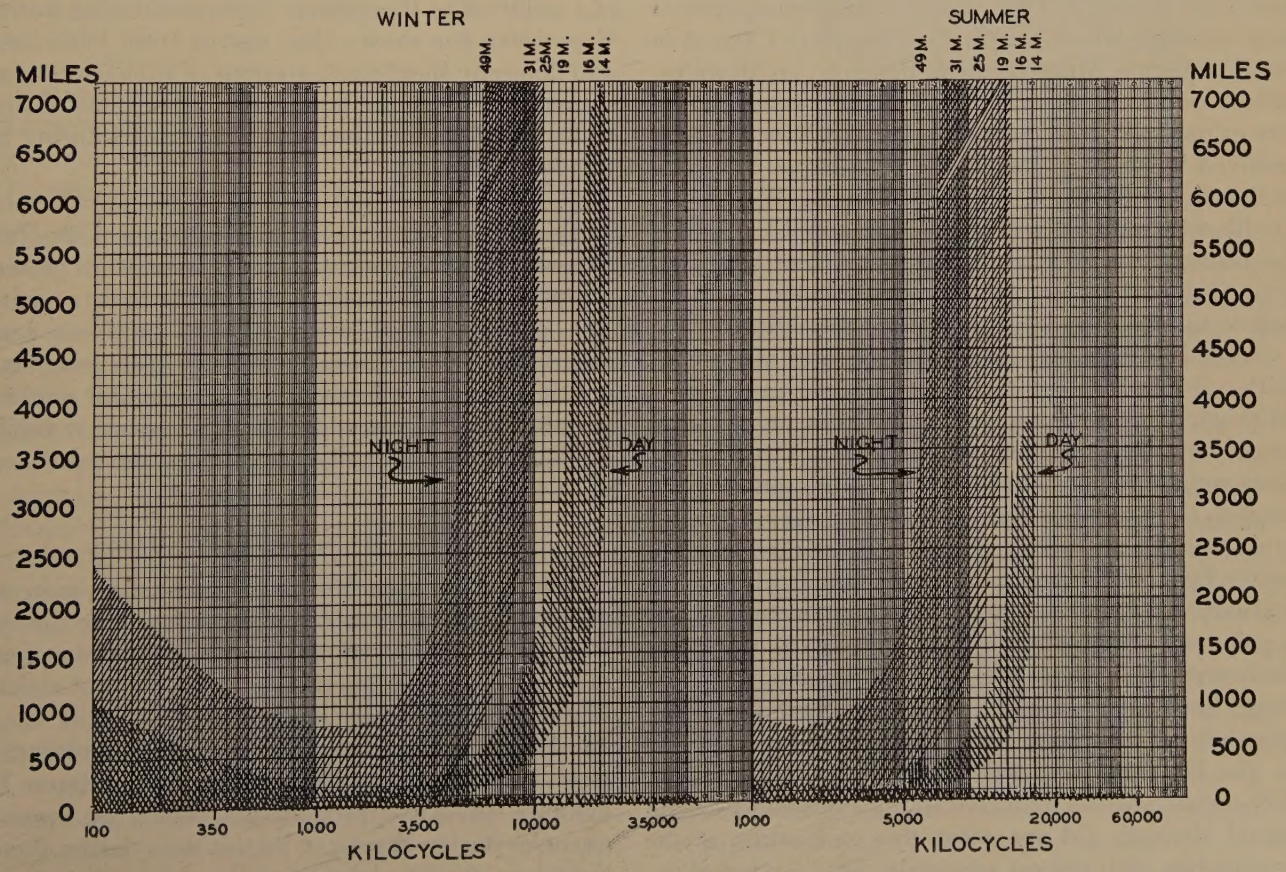

Fig. 10—Approximate distance range of radio reception. (Based on 10 microvolts per meter up to 2000 kilocycles decreasing to 1 microvolt per meter at 20,000 kilocycles, 5 kilowatts transmission.)

TABLE II

RESULT OF OBSERVATIONS WITH 9-TUBE, 5-BAND RECEIVER IN NORTHERN MICHIGAN

Approximate Time	Number of Strong Broadcast Stations	Total Number of Strong Short-Wave Stations	16-Meter Stations	19-Meter Stations	25-Meter Stations	31-Meter Stations	Location Best Short-Wave Stations
A.M. 8:30– 9:00	13	3	1			2	Daventry—San Francisco—Mexico City
A.M. 11:00–11:20	16	2		1	1		Guatemala City—Boston
P.M. 12:50– 1:10	11	7		5	1	1	Rome—London—Guatemala City—Schenectady
P.M. 2:00– 2:10	6	7		5	2		Daventry—Boston—New York—Schenectady—Guatemala City—Berlin
P.M. 3:00– 3:10	0	7	1	3	2	1	Daventry—Boston—Cincinnati—New York—Rome—Schenectady
P.M. 3:30– 3:45	0	12	1	4	2	5	New York—Guatemala City—Daventry—Philadelphia—Schenectady—Boston—Rome

of this program that there is much material of general interest. For example, news broadcasts are given at frequent intervals throughout the day and evening and at convenient listening times. Antenna arrays of some of the 50-kilowatt European short-wave transmitters are directed at the North American continent for propaganda purposes and, consequently, are capable of laying down a good signal under optimum conditions of propagation. Aside from certain of the foreign-language broadcasts, it is obvious that considerable entertainment and educational material is included. In addition, there are many American stations operating in the international short-wave bands which carry either network programs or programs intended for Central and South America. These stations, in general, operate into antennas which are directed at either Central or South America. Many of them, however, lay down excellent signals in many sections of the United States. For example, Boston, Schenectady, and New York are received in many places in the Mississippi Valley as well as in the Southeast. Stations from the West Coast are likewise received well in the Mississippi Valley and occasionally on the East Coast. In addition, programs from Guatemala City and Buenos Aires lay down excellent signals in many parts of the United States.

Table II illustrates briefly some results of a check with a 9-tube, 5-band auto radio receiver in northern Michigan. These checks were made from time to time as the auto was being driven north from Detroit. The tests were made on November 19, 1940, and were intended as routine observations on a representative cross-country trip which would at times carry the observer beyond the service area of standard broadcast stations. At 3:00 P.M. the observations were made at a point some 250 miles north of Detroit. At this point there were no broadcast signals sufficiently above the noise level to be usable. Excellent short-wave programs from Daventry, Rome, and a number of points in the United States were received in this vicinity. These observations indicated that serious variations in signal strength did not occur due to location of the automobile with respect to terrain. This section of the country was rolling and in places thickly wooded.

In January, 1941, some field-strength checks were made in the vicinity of Fort Wayne, Indiana, during the early afternoon. These checks were made on a 9-tube, 5-band auto receiver which had been calibrated with a reference oscillator. The collector consisted of a 96-inch cowl whip-type antenna. These checks indicated that field strengths sufficient to deliver 50 to 200 microvolts to the antenna terminals of the receiver were not uncommon. These observations indicated that under ordinary conditions of passing traffic, etc., field strengths sufficient to deliver 20 microvolts to the receiver input yielded quite acceptable programs. Signal strengths of Boston and Schenectady short-wave stations were sufficient to deliver signals of the order of 1 millivolt to the receiver terminals during this test. Experience has shown that signals from both foreign and domestic short-wave stations of sufficient strength to override the noise level under conditions of heavy traffic in urban locations are not unusual during both the day and night.

Thus, it appears that short-wave reception offers an additional service for those large portions of the United States in which broadcast transmission is not presently adequate for satisfactory automobile radio reception. The designer of automobile radio receivers can ill afford to overlook these possibilities. A receiver meeting this need must be carefully designed for maximum performance. When so designed, however, it fulfills a fundamental need and hence offers the user a new service.

CHOICE OF SHORT-WAVE BANDS

Reference to Figs. 5 to 10 indicates that, in general, program service should be available from the European stations in the central and eastern United States on 16 meters during the morning hours. Choice of signals is limited to a relatively few stations including London and Berlin which, as a rule, offer excellent program material and very usable signal strengths. Optimum long-distance sky-wave propagation occurs at lower frequencies during the night than it does during the day. Signals in the 19-meter band, in general, become adequate about midday and through the early and some-

times late afternoon. Signals on the 25-meter band usually become usable in the afternoon and early evening. Usually, 31-meter signals are most satisfactory in late afternoon and at night particularly in the Mississippi Valley. In the United States the 31-meter band is probably the outstanding nighttime short-wave band. Since the optimum wavelength is a function of time as well as distance, many programs on the 19-, 25-, and 31-meter bands are duplicated and either broadcast simultaneously or rebroadcast at appropriate times of day on all three of these bands. London and Berlin as a rule offer the most interesting foreign program material on all bands. Signals in the 49-meter band are useful in the continental United States at nighttime. In general, the program material is identical to that available at the same time in the 31-meter band and the standard broadcast band. There are, however, some Central and South American stations offering essentially local-interest programs in Portuguese and Spanish in this band. While this band is important in Central America and northern South America, it is not of as great importance in North America as the 31-meter band. In the particular receiver under discussion, four short-wave bands including 16-, 19-, 25-, and 31-meter bands were included.

Fig. 11 is a photograph of a receiver incorporating a standard broadcast band and four short-wave spread bands. Each short-wave band is spread to cover a few hundred kilocycles. In the four international short-wave bands, frequency coverage of each band is as follows:

16-meter band	17.7 to 18.0 megacycles
19-meter band	15.0 to 15.4 megacycles
25-meter band	11.6 to 12.1 megacycles
31-meter band	9.5 to 10.0 megacycles

The width of these bands is chosen to cover adequately the international short-wave broadcast assignments only. The small frequency spread in each spread band facilitates tuning since no band covers a range more than approximately half of the broadcast band. It is fortunate that the assigned bands are relatively narrow since international short-wave stations do not always operate with the same high degree of technique as American stations in the standard broadcast band. Interruptions to programs are more frequent and the commercial aspects of most short-wave programs do not require the same careful attention to the time of starting and stopping of programs or pauses in program compared to standard broadcast practice.

Circuit Considerations

Two basic electrical systems may be considered in the design of any multiband receiver. The first is the conventional single superheterodyne or double-detection receiver. The second is the double-superheterodyne or triple-detection receiver using the broadcast receiver as the first intermediate-frequency amplifier. The latter system was widely used during the early steps of development of combined broadcast and short-wave receivers for home use. Converters of this type were often sold as adaptors for use on standard broadcast household receivers. Two forms of the double-superheterodyne system were considered in the design of the receiver shown in Fig. 11; one in which the short-wave pre-amplifier is variably tuned and the other in which it is fixed-tuned with a given pass band. The variably tuned system enables the designer to control the band spread at will but necessitates either a second tuning control or a complex mechanical arrangement to preset and then disconnect the broadcast control while short waves are tuned. The use of a dual-control system is obviously objectionable from the consumer's standpoint. The alternative is likewise undesirable because of its complexity.

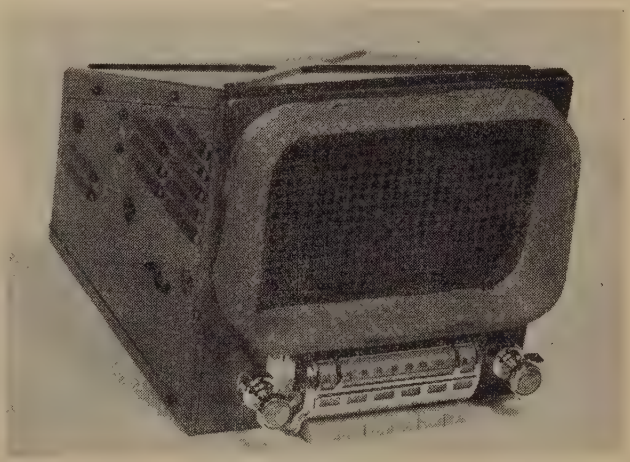

Fig. 11—Front view of 9-tube, 5-band auto radio receiver.

The fixed-tuned triple-detection receiver, on the other hand, can be arranged to pass a given, but not completely controllable, band of frequencies, which are then converted by a fixed oscillator to frequencies in the broadcast band. The signals can then be tuned by means of the broadcast receiver tuning gear. Thus the total frequency coverage and the tuning range on each short-wave band are identical to those of the broadcast band. Of the approximately 1000 kilocycles of coverage, however, the short-wave preselectors can be fixed-tuned to pass only an arbitrary band of the order of 200 kilocycles, determined by the Q's of the fixed-tuned circuits. Thus, the total coverage per band is fixed, and only an arbitrary portion of this fixed amount has usable sensitivity. Such a system is obviously very inflexible in this respect.

Both of the triple-detection arrangements tend to offer somewhat greater image rejection per tuned stage than does the double-detection receiver due to the higher intermediate frequency since the broadcast-frequency setting of the tuning control is the first intermediate frequency. On the other hand, the problem of spurious responses, tweets from the two oscil-

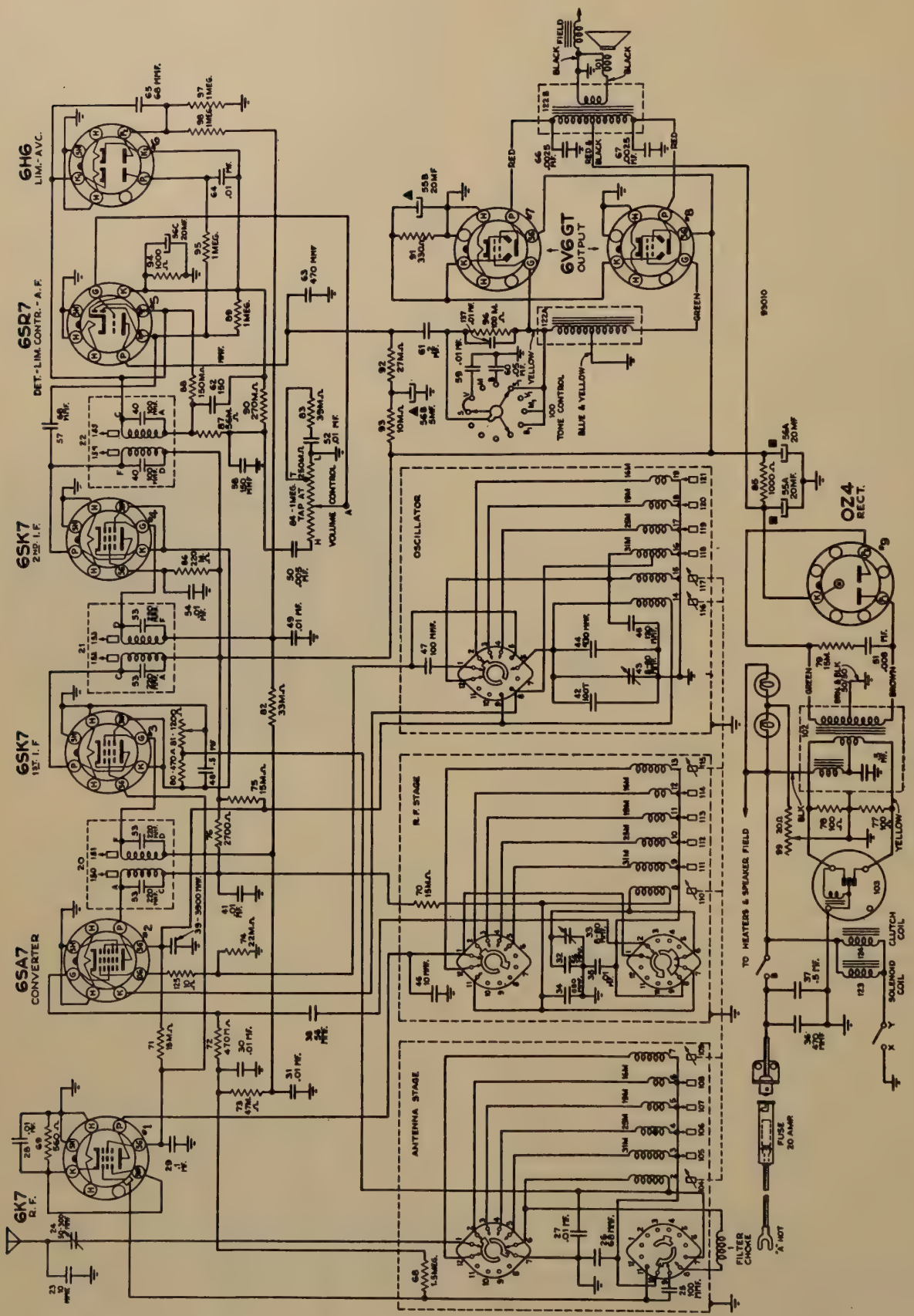

FIG. 12—Circuit schematic of 9-tube, 5-band auto radio receiver of Fig. 11.

lators of the triple-detection system, and fixed beats due to the ever-present strong broadcast signals coming through the limited attenuation of the short-wave preamplifier system, are problems of the first magnitude. The last-mentioned problem was met in early adaptors by instructing the operator to tune the broadcast receiver to a point in the band where no broadcast stations were received. This obviously renders impossible the use of a fixed dial calibration and, consequently, is unsatisfactory for modern auto receiver use. If this problem of fixed broadcast tweets were to be met directly, a broadcast attenuation of the order of 120 decibels or better would appear to be necessary.

The above paragraphs indicate only a few of the problems involved in determining the proper type of short-wave system to be used. A careful analysis of these problems indicates that the double-detection receiver using fully tuned radio-frequency and antenna circuits in an orthodox manner on all bands, best meets the over-all performance requirements. Such a system can be designed to be equal or superior to the triple-detection receiver in practically all respects. It can be made completely free from the possibility of broadcast interference. It allows freedom in control of dial arrangement and a choice of band coverage. For these and other reasons, the double-detection principle was used in the radio instrument discussed herein.

Fig. 12 shows the circuit diagram of the receiver of Fig. 11. This receiver employs 9 tubes and a non-synchronous vibrator. The receiver basically contains one 6K7 tuned radio-frequency stage on all bands, one 6SA7 converter, two 6SK7 intermediate-frequency stages, one 6SR7 functioning as a second detector, first audio-frequency stage, and noise-limiter control, one 6H6 noise-limiter automatic-volume-control tube, and two 6V6GT output tubes in push pull. A three-circuit permeability tuner is employed on all bands. An unorthodox construction has been developed for the radio-frequency system which mounts all of the essential components pertinent to a single stage within a single shield can. Each coil unit contains a broadcast-band tuning coil, a short-wave spread-band tuning coil, and fixed trimmed shunt coils for each of the spread bands. The use of the separate coils for each short-wave band provides independent control of alignment and dial calibration on all bands by direct alignment methods and without compromise. The unit type of construction makes it possible to bring a minimum number of leads out of each coil unit. This results in a minimum of intercoupling in a layout which is very closely limited in space. It also provides a minimum amount of interference pickup from the power supply of the receiver or from the ignition system of the automobile. The broadcast coils and the short-wave spread-band tuning coils are both permeability tuned. The broadcast coils are tuned with high-permeability iron cores. The short-wave spread-band tuning coils are tuned with cores of low-permeability material and are designed, in conjunction with the shunt short-wave coils, to provide the desired band spread on all bands. The short-wave circuits are aligned to the dial on each band by means of trimmer cores in each of the short-wave shunt coils. The dial calibration is essentially linear on all short-wave bands.

The antenna circuits are all closely coupled to the low-capacitance whip-type car antenna through a single series trimmer which is arranged to provide exact alignment of the antenna circuit on all bands. On the broadcast band a high-frequency filter choke is switched into the grid circuit of the radio-frequency tube to minimize ignition interference picked up by the antenna. On the short-wave bands, the short-wave coils are arranged in series between the antenna and the grid of the tube for the same purpose.

The coils of the radio-frequency amplifier stage are also fully tuned and tracked with the oscillator on all bands in a manner similar to that employed in the antenna stage. In the radio-frequency stage the broadcast coil is arranged to provide a maximum of image attenuation, and the gain is low to aid in obtaining the desired sensitivities on all bands. On the short-wave bands, the radio-frequency amplifier circuits are arranged to provide a maximum gain and image attenuation.

The oscillator is of the modified Colpitts type on the broadcast band and of a Hartley type on the short-wave bands. The oscillator of the broadcast band is designed to track the signal-frequency circuits by means of a variable-pitch winding of proper diameter. The oscillator coils of the short-wave circuits are arranged to tune with a relatively large fixed capacitor which has a proper negative temperature coefficient to obtain maximum oscillator stability under conditions of varying temperature. A single tapped coil operating in shunt with the tuning coil comprises the 31-meter-band tank inductance, while on the higher-frequency bands a third coil of proper value is shunted across these master coils. Fig. 13 is a cross-sectional view of the radio-frequency coil assembly just described.

Receiver Performance

The performance resulting from the use of these circuit arrangements is shown in the following figures. Fig. 14 shows the curves of sensitivity and noise percentage at a 1-watt output level with a 30 per cent modulated radio-frequency carrier input applied from a low impedance through a dummy antenna of 33 micromicrofarads capacitance and 33 micromicrofarads shunt capacitance. The curve of Fig. 15 shows the image ratio at various frequencies in the broadcast band. Table III gives the sensitivity and per cent of noise at 1 watt output level and image rejection data for the various short-wave bands.

A 455-kilocycle intermediate frequency is used since the 260-kilocycle intermediate frequency commonly

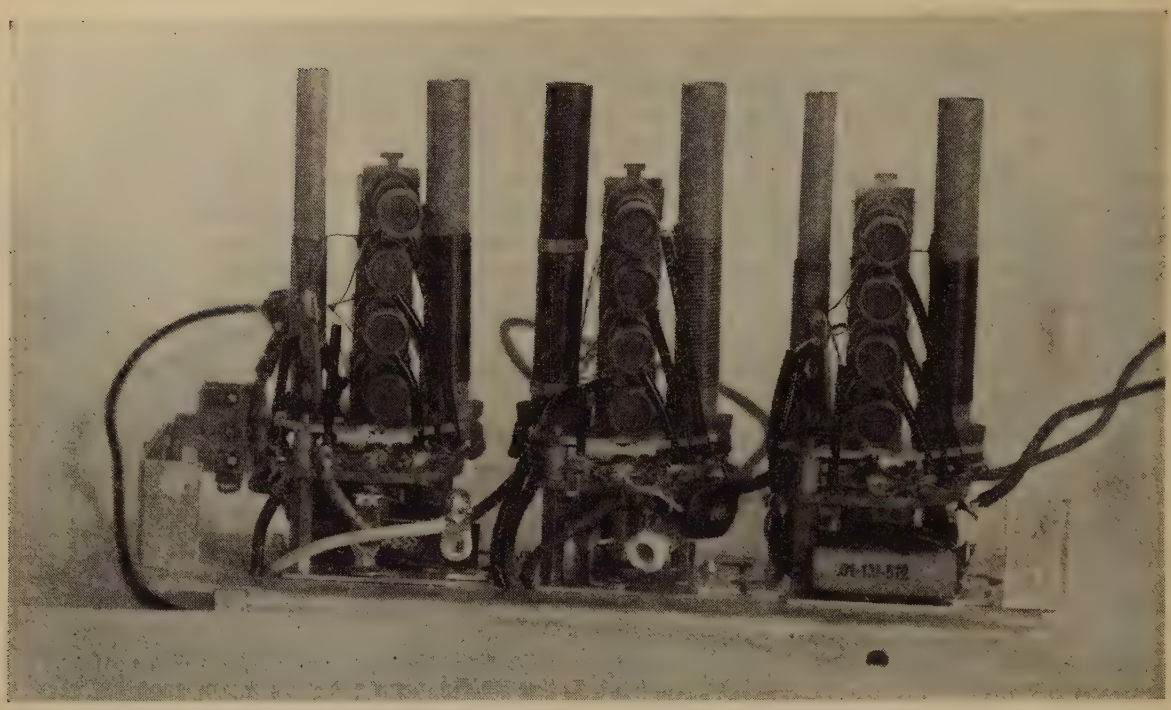

Fig. 13—Radio-frequency coil assemblies of 9-tube, 5-band auto radio receiver.

used in auto receivers is too low for satisfactory short-wave performance. Two intermediate stages are used to obtain the desired selectivity and gain. Sensitivity

TABLE III
SHORT-WAVE PERFORMANCE INDEXES

Band	Sensitivity	Per Cent Noise	Image Rejection
31	0.8	33	85
25	1.0	25	41
19	1.6	16	34
16	1.8	10	20

control in the broadcast band is obtained by means of a variable resistor in the intermediate-frequency-amplifier cathodes. Thus all receivers can be adjusted to the same broadcast sensitivity at the factory. The cathodes of the intermediate-frequency amplifiers are switched by means of contacts on the range switch so as to increase the intermediate-frequency gain on the short-wave bands. This control, together with the design used in the radio-frequency-amplifier stage, results in the desired sensitivity on the various bands. The over-all selectivity characteristic is shown in Fig. 16.

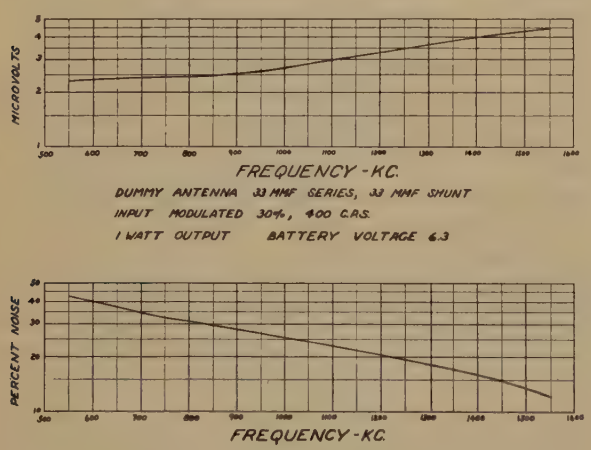
MICROVOLTS
FREQUENCY - KC.
DUMMY ANTENNA 33 MMF SERIES, 33 MMF SHUNT
INPUT MODULATED 30%, 400 C.P.S.
1 WATT OUTPUT BATTERY VOLTAGE 6.3
PERCENT NOISE
FREQUENCY - KC.

Fig. 14—Sensitivity and noise characteristic of 9-tube, 5-band auto radio receiver.

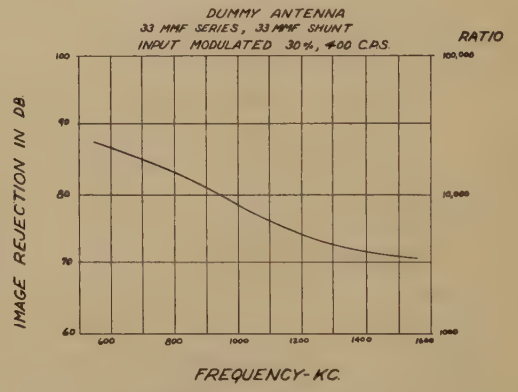

Fig. 15—Image rejection, broadcast band, 9-tube, 5-band auto radio receiver.

Noise Limiter

Ignition noise is of much more importance in the short-wave bands than in the standard broadcast band. This is due to the lower average signal strength in the short-wave bands and to the fact that energy radiation from auto ignition systems is greater at short waves. There are a number of possible noise-limiter circuits which could be used effectively in short-wave auto receivers. These circuits must be designed to operate without manual adjustment for any signal input to the receiver. They should be, and in general are, most effective on weak signals and between stations. Limiters, in general, consist functionally of an automatic control system and a limiter proper which either opens an audio-frequency circuit, short-circuits an audio-frequency circuit, or balances noise impulses

out of the audio-frequency system. In this way, noise impulses may be limited to an arbitrary percentage of modulation determined by the values chosen for the circuit constants. Limiters operate most effectively on those types of noise which consist of sharp repetitive impulses. Much man-made interference is of this nature, in particular the sharp wave fronts radiated by automobile ignition systems.[8] In the case of smoother noise such as tube "hiss," thermal agitation, etc., the noise limiter is, of course, of no practical value since the limiter and control system cannot distinguish between this type of noise and the desired signals. Consequently, it is to be expected that as the character of

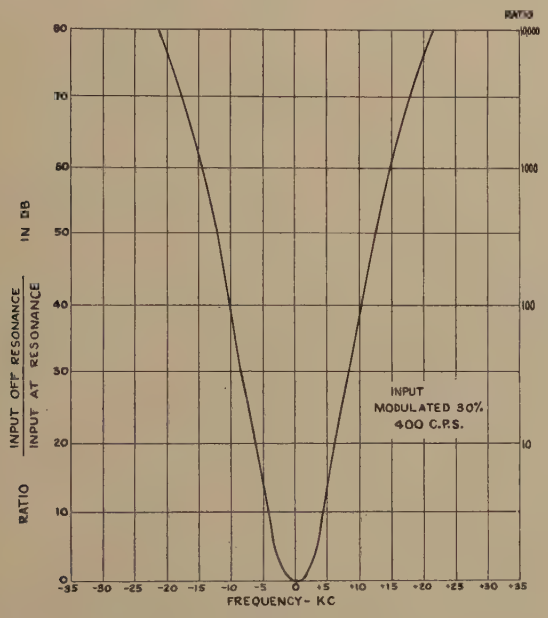

Fig. 16—1000-kilocycle over-all selectivity of 9-tube, 5-band auto radio receiver.

the noise changes from sharp repetitive impulses to "smooth" noise, the limiter becomes less effective.

From the customer's viewpoint, the limiter performs best when it is most needed. It will greatly reduce noise between stations without introducing distortion on weak signals. The limiter is also effective on weak signals and often makes it possible to use weak signals which would otherwise be of no use due to noise interference. As signals become stronger, the limiter becomes less effective, but limiter action is not then as important.

The limiter circuit incorporated in the receiver under discussion is such that limiter action begins at 100 per cent modulation level regardless of the signal input or the signal level at the second detector. This automatic control is provided by means of a diode in the 6SR7 to which the signal is fed from the primary of the final intermediate-frequency transformer. The limiter proper consists of a reverse diode, in parallel with the second-detector diode, which conducts on noise peaks exceeding 100 per cent modulation in such a way that the noise impulses balance out in the second-detector diode load resistor. The limiter is very effective in reducing interchannel noise as well as repetitive types of noise on weak carriers where noise reduction is most needed.

In a broadcast-band automobile receiver using a relatively short rod antenna,[9] the input to the receiver may persistently drop to low values. This is particularly true near the edge of the service area of stations, when passing through hilly country, cities, steel bridges, underpasses, etc. It is also true, of course, in the reception of international short-wave broadcasts. For this reason, the automatic-volume-control system used in the receiver shown in Fig. 11 departs slightly from the conventional automatic-volume-control arrangement in that the amount of delay in the system was intentionally kept low in order to maintain automatic-volume-control action at low signal inputs. It can be seen from curve *A* of Fig. 17 that automatic-

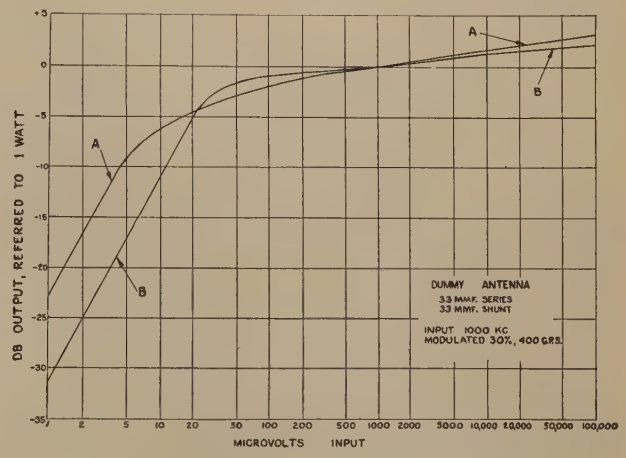

Fig. 17—Automatic-volume-control characteristics.

volume-control action begins at approximately 5 microvolts input on the broadcast band.

This curve represents the automatic-volume-control characteristic of the receiver under discussion. Curve *B* is representative of the automatic-volume-control characteristic of the usual automobile receiver with high automatic-volume-control delay. In this characteristic, automatic volume control does not begin until inputs of approximately 25 microvolts have been reached. If a signal is being received which varies in strength from 5 microvolts to 1 millivolt, which is not unusual for short-wave reception, the audio-frequency output would vary some 10 decibels on curve *A* compared to approximately 18 decibels on curve *B*. This, of course, presupposes that the audio-frequency system does not overload and therefore does not alter the shape of the over-all characteristic. This particular case would therefore hold for the case in which the volume control was adjusted to normal listening level when the signal input was above the knee of the curve.

Above the delay point of the automatic-volume-

[8] L. F. Curtis, "Ignition disturbances," *Proc. Radio Club Amer.*, October, 1936.

[9] F. M. Colebrook, "The theory of receiving aerials," *Exp. Wireless*, vol. 4, p. 647; 1927.

control characteristic, the automatic-volume-control action obtained with high delay is somewhat superior to that obtained with a small amount of delay. In addition, the power sensitivity or overload curve of the receiver will rise much more steeply when a higher delay voltage is used. The power-sensitivity curve refers, of course, to the characteristic taken with wide-open volume control and with a 30 per cent modulated

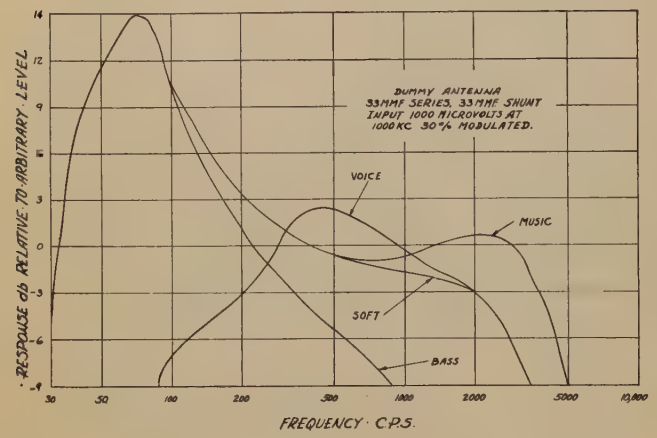

Fig. 18—Electrical fidelity of 9-tube, 5-band auto radio receiver for four tone-control positions.

input signal. This design was used after considering the possibilities of parallel-channel automatic-volume-control and other arrangements which give slightly flatter automatic-volume-control characteristics at higher signal input levels but only at a considerable increase in cost and space requirements.

The audio-frequency system of the receiver consists of a 6SR7 audio-frequency amplifier stage and a class A push-pull 6V6GT output stage driven through an interstage auto transformer. Fig. 18 shows the electrical fidelity characteristics for the four tone-control positions. The "music" position offers maximum range. In the "bass" position the high frequencies are reduced to a minimum. The "soft" position has maximum low-frequency response and medium high-frequency response. The "voice" position has medium high-frequency response and greatly reduced low-frequency response to give satisfactory fidelity on voice and on certain foreign programs in which the low-frequency response is greatly exaggerated as compared to the balance of programs in standard broadcasts.

Power-output-versus-distortion characteristics are shown on Fig. 19 for tube battery voltages of 6.3 and 7.4 volts. The latter is approximately the voltage maintained by the automobile voltage regulator. The power is fed through a conventional transformer into an elliptical speaker. The use of the elliptical speaker allows maximum size and power-handling capacity with a minimum height of the instrument panel. The power supply is of the conventional nonsynchronous-vibrator type[10] and utilizes an OZ4 cold-cathode rectifier. Filtering and by-passing are conventional. Two

[10] J. C. Smith, "Automobile receiver design," *RCA Rev.*, vol. 1, p. 94; April, 1937.

filter elements included in the power-transformer assembly and two by-pass condensers included on the *A* lead provide means for suppressing vibrator interference.

The sources of ignition interference as well as other electrical interference originating in the electrical system of the car are adjacent to the receiver and antenna. Consequently, it is necessary to deal with the induction field rather than the radiation field of the interference source. In this fundamental respect the ignition-interference problem differs from the problem of interference from passing cars. Ignition and other electrical disturbances originating in the car set up interference largely through the medium of shock excitation in the first tuned circuit. Because of this, it has been customary to incorporate in the antenna system filtering means to suppress the components of the disturbances outside the desired reception band. The practical design difficulties involved in incorporating effective filtering means increase as the frequency of the desired reception band increases. To reduce the disturbances within the desired reception band, it is necessary to operate directly on the sources of interference. The paths of interference currents consisting of the primary electrical system and the car's metal parts must be controlled in such a manner that the resulting coupling produces interference less than the level of first-circuit noise. In general, the electrostatic shielding of the hood and other adjustable metal parts of the car is less effective in the short-wave bands than in the broadcast band. General ignition-interference-reduction problems are however, similar in the broadcast and short-wave bands and frequently can be treated in a similar manner. The installation of the

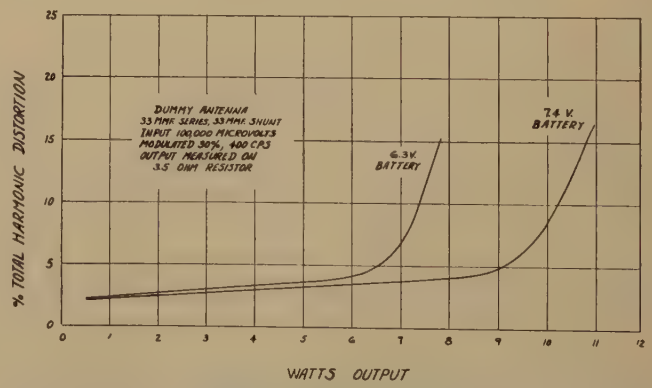

Fig. 19—Power output and distortion characteristics of 9-tube, 5-band auto radio receiver.

receiver described in this paper required redesign of the generator by-pass capacitor to reduce its impedance at high frequencies to a value comparable to that normally encountered in the broadcast band. Similar treatment of the voltage regulator was required.

Mechanical Design Considerations

One of the initial premises in the design of this receiver was compactness. It was necessary that it be a single-unit instrument complete in one case and ar-

ranged so that it would mount on the instrument panel in a conventional manner. It was necessary that the design be such that broadcast performance need not be compromised in order to incorporate short-wave reception. It was necessary to use the equivalent of a full 8-inch speaker. It was necessary to give considerable care to the accessibility of tubes and vibrator and to provide for their easy removal and replacement with the receiver installed in the car. For customer convenience, a solenoid-actuated push-button tuning mechanism includes 5 station push buttons in the assembly. Any one of the five positions can be set up on any frequency in any band with one hand and without the use of tools. Dial and control provisions are similar to those of conventional receivers except that only one position switch synchronized with the tone drum which labels the four tone positions, VOICE, MUSIC, BASS, and SOFT in fixed sequence. Each operation of the button advances the tone switch one position.

A dial-light dimmer control is provided with which the brilliancy of dial illumination may be varied to suit the individual. The knob for this control is mounted on the bottom of the receiver case and is accessible to the driver under the instrument panel. The receiver is conventionally mounted with a straphanger from a stud on the rear of the case and from the two control bushings on the front. The elliptical speaker, which is mounted inside the case and directly above the dial, operates through the grille mounted in the instrument panel of the car.

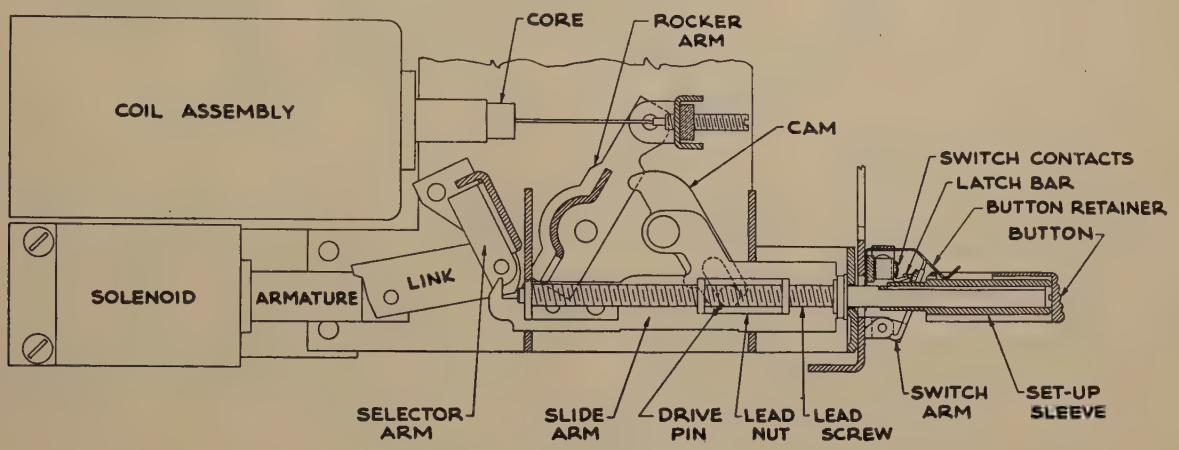

Fig. 20—Tuner cross section (at rest).

dial calibration is exposed at one time. Due to space limitations, it was necessary to add the four short-wave spread bands without increasing the case size over that of a conventional standard-band receiver. As shown in Fig. 11, the external appearance and size of the receiver closely resembles that of a standard broadcast instrument. The arrangement of dial and controls is symmetrical and compact and the control unit is designed to mount in the instrument-panel openings provided for a standard broadcast receiver. The knob on the left controls the power switch and volume while the knob and wing knob on the right are for manual tuning and range-switch operation, respectively. The chrome-plated die casting, mounted centrally between the two knob centers, houses the cylindrical multiband dial, the band-indicator drum, the tone drum, the call-letter clip, the tone-control push button, and 5 station-selector push buttons. The dial scale is viewed through the long central window. The small window on the right is the band-indicator opening and the one on the left is for tone-position indication. The call-letter clip is mounted immediately below the 3 windows. Below the call-letter clip are the 6 buttons, the one on the left being used for tone-control operation. The volume and on—off-switch operation is standard as is the manual tuning. The tone control consists of a four-

It was possible to house the complete receiver in a single case of the size and shape shown in Fig. 11, because of (1) the development of an elliptical speaker equivalent in characteristics and performance to an 8-inch circular speaker; (2) dial and tuning mechanism design permitting the lower extremity of the speaker cone housing to nest between the upper sections of the dial and tuner. This arrangement resulted in a front elevation of normal height; (3) permeability or inductance tuning for all bands; and (4) elimination of the range switch from the category of a large component by splitting it up into three sections, each mounted within the coil can of the circuit it controls.

Accessibility of tubes and vibrator was accomplished in the usual manner of mounting tubes and vibrator vertically and hanging downward from the chassis. A removable bottom cover provides accessibility to these parts with the receiver in position in the car.

Solenoid Tuner

From a mechanical viewpoint, the solenoid tuner and dial and range-control provisions are the features of most interest. The tuning mechanism is a 5-position solenoid-actuated unit complete with manual drive, solenoid, clutch, pointer drive, with provisions for setting up stations with one hand, without a tool.

Fig. 20 is a front-to-rear vertical cross-sectional diagram showing the elements of the tuning mechanism in the rest position. To operate any push button, the button for that position is advanced 3/16 of an inch. This operation advances the slide arm sufficiently to place the hooked end in the path of the selector arm; pivots the switch arm from its forward-leaning position to the vertical, at which point the switch contacts meet; and raises the switch-arm latch bar to a position in which it clears the switch arm. When the switch contacts meet, the resulting current energizes the solenoid and clutch. The clutch releases the rocker arm and the solenoid pivots the selector arm which engages the advanced slide arm and further advances it until both cam contact points meet the rocker arm. When one cam contact point meets the rocker arm, the rocker arm starts to turn and continues until stopped by the opposite cam contact point. The position of the cam is set in the station-setup procedure described below. The rotation of the rocker arm pushes or pulls (depending upon direction of rotation) the cores in or out of the coils to the preset position as determined by the cam setting. Fig. 21 shows all elements of the mechanism in positions resulting from the above procedure and action.

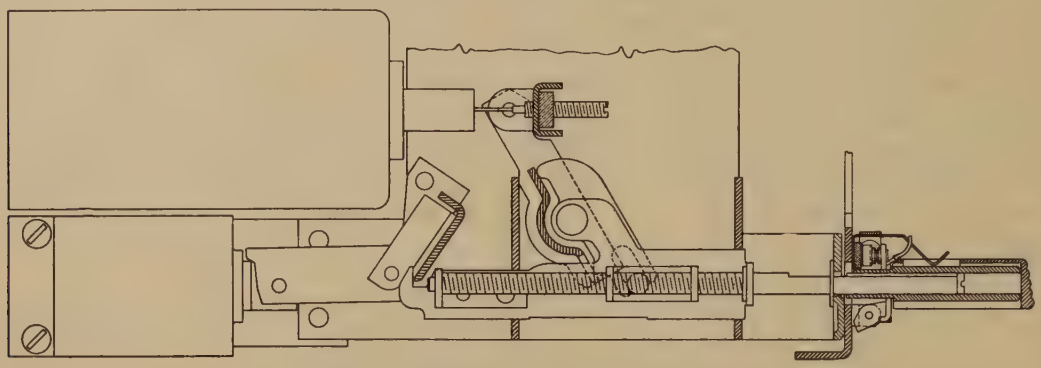

Fig. 21—Tuner cross section (actuated).

Refer to Fig. 20, and consider the slide-arm assembly as a unit; it consists of the arm, the cam pivoted thereon, a lead nut, lead screw, and setup sleeve. The lead screw drives the lead nut forward or backward. Fastened to the lead nut and passing through a clearance slot in the slide arm is a drive pin which engages in a slot in the lower section of the cam. Movement of the lead nut rotates the cam about its pivotal center through an angle of 60 degrees (30 degrees either side of the vertical). The outer end of the lead screw is provided with a flat. The setup sleeve has on its inner end a short flattened section which engages the lead screw flat and serves as a wrench. The sleeve slides freely in or out on the lead screw.

The setup of any one of the 5 positions is accomplished as follows: The button for the position to be set is first removed. (The buttons are held in position and prevented from coming off during normal operation by means of a retaining spring.) This spring is displaced and releases the button when it is pulled outwardly. Removal of the button exposes the setup sleeve. When the setup sleeve is pushed in, the results are the same as those caused by pushing a button with one exception. Since the button is not in position to lift the latch bar clear of the switch arm, the switch arm is latched and held in the vertical position, as shown in Fig. 22. This sustains a closed circuit for the clutch and solenoid, instead of the momentary circuit resulting from push-button operation. The cam is thus held against the rocker arm as shown in Fig. 21. For convenient operation the setup sleeve may be pulled out to its full extension and turned in the direction desired. Turning the sleeve pivots the cam which, in turn, rotates the rocker arm. Essentially, a manual tuning operation is performed. When the selected station is tuned, the

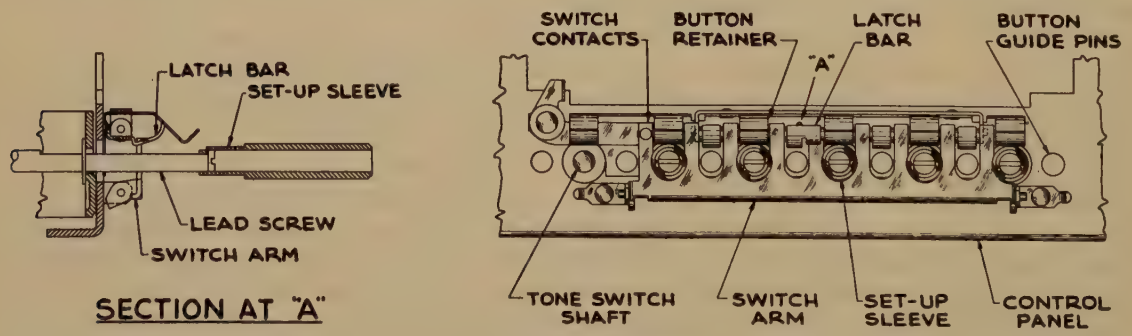

Fig. 22—Switching mechanism.

button for that position is replaced. The button trips the latch and returns all elements to the rest position. Subsequent use of the button tunes the receiver to the preset frequency.

Range Switch

Fig. 23 is a schematic diagram showing the three range-switch sections and all elements involved in range-switch and dial operation. In this illustration all bearings, support members, shields, coils, and other items not involved in the fundamental drive system are omitted. The manual-tuning and range-switch

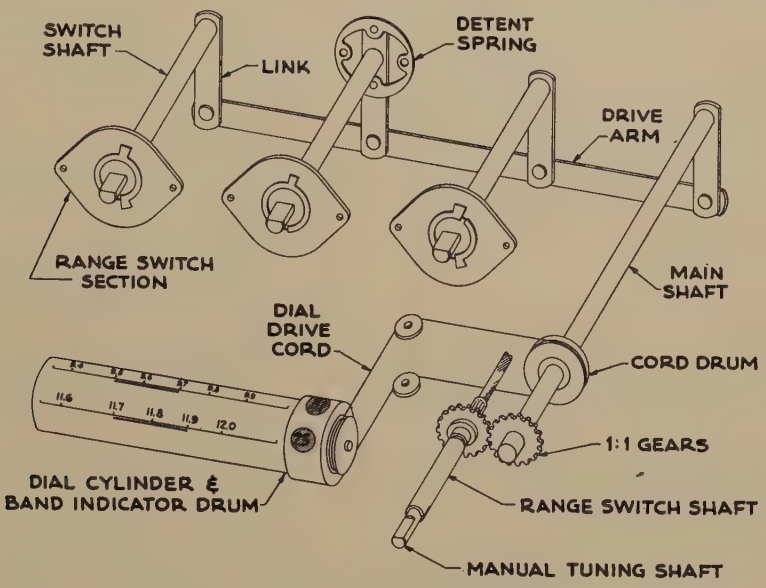

Fig. 23—Dial and band-switch schematic.

shafts are coaxial, the range-switch shaft being the outer or hollow shaft. To allow clearance for the manual-tuning shaft, the main range-switch shaft is offset through a pair of gears at a 1:1 ratio. Beyond the gear on the main shaft is a cord drum which operates the dial in synchronism with the range-switch sections. The cord drum is adjustable radially on the shaft to provide for alignment of dial calibrations in the dial window. The main shaft extends toward the rear of the receiver and engages the link-and-arm assembly. The links extending from the three switch-section shafts and main shaft are connected together with a common drive arm. On the center-switch-section shaft is shown the detent spring which accurately registers the entire drive system in each of the five positions as the wing knob is operated. The diagram is shown with the system in the mid-position. There are five positions, one for each band covered by the receiver. The switch rotation between positions is 30 degrees and since the wing knob is connected into the system in a 1:1 ratio, its rotation from one band to the next is also 30 degrees. In order to spread calibrations on the dial cylinder and to use effectively the available cylinder area, a 2:1 ratio is used, resulting in a 60-degree dial rotation between bands.

Conclusion

It is believed that the development of this receiver represents a noteworthy step in automobile radio receiver design. It is already apparent that it has opened up markets heretofore not available in this field because of adverse conditions of reception in many localities. Moreover, it has brought to many auto radio users in certain localities daytime or in some cases even nighttime reception for the first time.

It has long been realized that adequate coverage of the entire United States using the present broadcast assignments is not likely to be realized for some time in the future for economic and technical reasons. Properly located long-wave high-power broadcast stations similar to those in use in Europe might offer one solution to the problem of coverage of the United States. However, a project of this kind that would provide adequate choice of programs would appear to be faced with serious economic handicaps and in addition would require the assignment of frequencies now in use for aircraft navigation. The case for long-wave coverage of the United States has been ably presented by Eckersley,[11] Wenstron,[12] Aiken[13] and others. The validity from a technical standpoint of the case for long-wave coverage is not questioned. It is believed, however, that a more economical approach and one which could be more nearly self-supporting might be set up in the present international short-wave bands. Such a system might be made to serve admirably many of the areas not now properly served by standard broadcast sta-

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tions. One possibility might be the employment, with nondirectional antennas, of present American international short-wave stations on frequencies not desirable for transmission to another continent. For example, when the 16-meter band was in use for intercontinental broadcasting, the 19-meter or 25-meter band might be used for local coverage within the United States. Obviously, a directive antenna with the signal directed toward some specific area might have certain advantages in cases of this sort.

Such a system could, by means of a relatively few stations, provide a badly needed daytime service in those large areas not now adequately served with radio programs. This system, in conjunction with short-wave automobile receivers, such as the one described herein, should overcome one of the greatest handicaps that now face the further development of automobile radio use, that is, present lack of satisfactory daytime signals in other than metropolitan areas, and as a result might provide a great stimulus to the use of automobile receivers.

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Recent Developments in the Electron Microscope*

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Summary—A description is given of the limitations in resolving power of microscopes indicating the reason for the 50- to 100-times greater resolving power of the electron microscope over the optical instrument. This is followed by a treatment of the basic principles involved in the electron microscope. A detailed description is given of the newly developed RCA electron microscope which, although equal or superior in performance to any previous microscope, occupies a much smaller space, is completely self-contained, and can be readily handled by an inexperienced operator. Examples of the work done by this new instrument are shown. The extremely compact power-supply system for the microscope is described. The reduction in size is obtained chiefly by the use of a radio-frequency-actuated rectifier system. By the use of radio frequency and an especially designed voltage divider, excellent stability of the order of 0.004 per cent of 60 kilovolts output at 0.5 milliampere drain is obtained.

Introduction

DURING the latter half of the last century, Abbe demonstrated the diffraction theory of microscopic vision, and in so doing discovered the limitations imposed on the resolving power of any microscope by the wave nature of the radiation used. A mathematical investigation of the problem revealed that a microscope cannot produce separated images of two points which lie closer than d, when

$$d = \frac{0.61\lambda}{\mu \sin \alpha}, \qquad (1)$$

where λ = the wavelength of the illumination used,

μ = the index of refraction of the medium in which the object is situated,

α = the maximum angle which a ray leaving the central point of the object and entering the objective may make with the optic axis of the system.

Since the maximum possible value of $\sin \alpha$ is 1, and since no optical media with refractive indexes greater than 1.7 which are transparent for visible or ultraviolet light have been discovered, we can see that the only possible way to make a really large reduction in d is to make λ extremely small. However, attempts to extend the resolving limit of the microscope by the use of shorter wavelengths have been limited by the apparently fundamental lack of suitable refractive materials. By the use of ultraviolet light and extremely precise quartz and fluorite lenses, research workers have succeeded in obtaining a limiting resolution of slightly better than 1000Å, but this is only $2\frac{1}{2}$ times better than the limiting resolution obtainable with an ordinary oil-immersion objective used with visible light, and still requires that the range of structures between 1000Å and atomic dimensions be investigated by other methods.[1]

The association by de Broglie of a periodic phenomenon with moving particles of atomic dimensions and the mathematical demonstration by Busch of the similarity between axially symmetrical magnetic and electric fields and spherical lens surfaces, in regard to their action on charged particles and light rays, respectively, constitute the fundamental conceptions which made possible the development of a microscope using field lenses and a corpuscular type of radiation. The knowledge which has already been gathered regarding the production and control of electron beams and the immediate development of electron lenses for cathode-ray tubes for oscillography and television, taken in conjunction with the fact that the wavelength associated with a beam of electrons accelerated by a fairly high potential is extremely minute, made the electron beam the natural choice for use as the radiation in a corpuscular microscope. This idea was first investigated about eight years ago,[2] and since then has been the subject of considerable interest in various physics laboratories all over the world.[3–7] All of these electron microscopes, including an earlier model constructed by L. Marton[8] in the RCA Laboratories, were very large and complicated, and required a specially trained operator to manipulate them. Thus these instruments were only suited for a physics laboratory, rather than for use as a general research tool. The design of the instrument here described was undertaken to overcome the objectionable features of the earlier models, and to produce a small, compact electron microscope capable of extremely high resolving power, which could be operated by any competent laboratory technician, so that it can meet the requirements of any industrial or scientific research laboratory.

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[1] In the following discussion, the term "useful magnification" will always mean that magnification which is necessary to increase the size of details, the dimensions of which are equal to the resolving limit of the system used, to the point where they are distinguishable by the unaided eye (i.e., 0.2 millimeter). Thus the limiting useful magnification of the ultraviolet microscope is approximately 2000 times.

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Since, in the following discussion, the electron microscope will be described by a direct comparison with the analogous parts and functions in the light microscope, it is desirable to point out a few of the fundamental features of the latter instrument.

The modern light microscope usually consists of a system of solid lenses, which, with the exception of

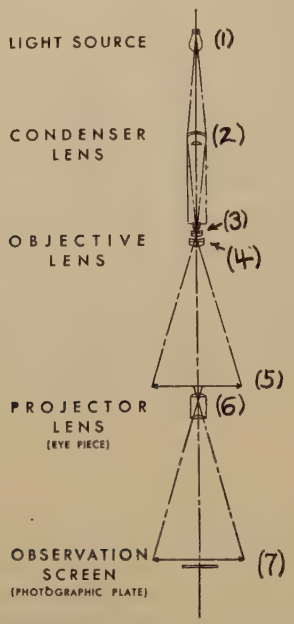

Fig. 1—Schematic diagram of light microscope.

those of the illuminating system, are kept in precise alignment by accurately machined holders. A schematic diagram of the system of lenses used in the light microscope is shown in Fig. 1. The light source usually consists of an incandescent lamp (1), which may be specially constructed so that the illumination originates from a small region which is either spherical or approximates a point. Light leaving this source enters the substage condenser (2), either directly or after being reflected by a suitable mirror. In addition to condensing the illumination on the specimen (3), the substage condenser serves as a means of controlling the intensity and angular aperture of the illuminating beam. It may be fitted with an iris diaphragm to provide one method by which this adjustment may be made. On passing through the specimen, the beam of light undergoes considerable change, the nature of which varies over the plane of the specimen according to the properties at its different points. The properties of the specimen which determine the degree and nature of the change produced are (a) variations in the index of refraction over the specimen; (b) differences in the selective absorption; (c) differences in the scattering power of the object for the light used; (d) conditions of reflection from variously orientated surfaces in the specimen; and (e) the size of smaller particles and structures with the associated diffraction effects. The action of the objective lens (4) is such that the disturbed light beam leaving the plane of the specimen is reproduced on an enlarged scale in the image plane (5), thus forming what is known as the first image. Owing to the diffraction defect which has already been mentioned, and the aberrations introduced by the objective lens, the reproduction is not absolutely accurate in its finest detail, though the image produced in this manner contains all the information

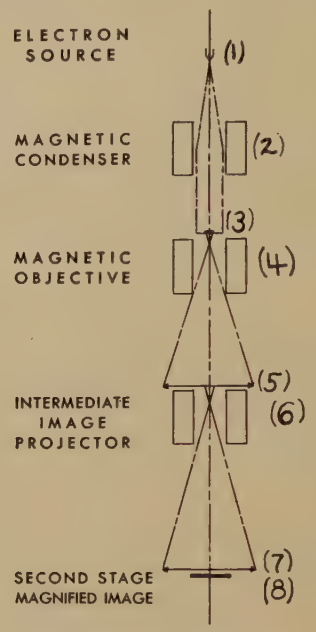

Fig. 2—Simplified schematic cross section of magnetic electron microscope.

that will be obtained with a microscope using a given objective. However, such information is not yet in a form which can be observed by an unaided eye because the enlarging action of the objective alone is not sufficiently great. For instance, in the case of high-power, oil-immersion objectives, where the magnification is usually of the order of 100 times, the image of those structures whose dimensions are less than 2 microns cannot be distinguished by the eye, even though they exist in the first image. This is overcome by the use of a second stage of magnification which may be accomplished by any means that does not lose any of the detail of the first image. Fig. 1 shows an eyepiece (6) being used as a projection lens to form a further magnified image of the specimen on an observation screen or photographic plate (7). The over-all magnification should be such that the finest details present in the first image are now visible to the unaided eye; i.e., for the best ultraviolet microscope, the magnification should be about 2000 times.

Fig. 2 shows a very much simplified schematic cross section of a magnetic electron microscope. The arrangement of the component parts is seen to be quite similar to that of the light microscope, the various light components being replaced by the corresponding electron components. The light beam is replaced by a beam of fast-moving electrons, and the glass lenses are replaced by magnetic-field lenses which are essentially the magnetic fields of short solenoids through

which a very constant current is flowing. In order to concentrate the magnetic field into a small region about the axis of the solenoid, the latter, with the exception of a narrow gap in the central opening, is completely encased in a symmetrical iron sheath.

Electrons leaving a heated tungsten filament (1), and accelerated along the axis of the electron microscope by a high potential, form an electron beam which, after passing through the condenser lens (2), is used to irradiate the specimen (3). The magnetic condenser lens (2) corresponds to the substage condenser of the light microscope, in that it performs the same function. A means of varying the current flowing in the winding of the condenser lens replaces the adjustment of the iris diaphragm in the light condenser as a means of controlling the intensity and angular aperture of the irradiating beam. As in the case of the light beam passing through the specimen in the light microscope, the electron beam suffers a change on passing through the specimen in the electron microscope. However, the nature and cause of this change and its action in producing an electron image of the specimen are somewhat different, and may prove to be less complex in the case of the electron microscope. The change introduced in the electron beam by its passage through the specimen depends (a) on the scattering and absorbing power of each point of the specimen for electrons of the velocity used, and (b) on the size of the smaller particles and structures and the associated diffraction effects. The action of the strong, axially symmetrical magnetic field of the objective lens (4) on the electrons leaving the specimen is such as to produce in the first image plane (5) a distribution of electrons which is an enlarged reproduction of a certain distribution of electrons in the plane immediately below the specimen. This certain distribution is the one obtained by considering only those electrons which leave the specimen in paths of which the divergence from the axis of the objective is less than a predetermined maximum value. The accuracy with which this distribution is reproduced in the image plane (5) is limited by the diffraction defect and the aberrations of the objective, as in the case of the light microscope. However, as will be seen below, the accuracy of reproduction in the case of the electron microscope is much greater than the corresponding accuracy in the case of the light microscope. The magnification produced by the electron objective is also quite low, being about 150 times in the microscope which is described in the following section, so that the first image contains a large amount of information which cannot be observed by the unaided eye. Moreover, the detail which is contained in the first electron image is so fine that total magnifications of the order of 100,000 times are necessary before all of that detail is large enough to be visible to the eye. Once again, the secondary stage of magnification required to realize this high final magnification may be accomplished by *any* method which does not destroy or lose any of the information present in the first image. In practice it has been found most satisfactory to use a magnetic projection lens (6) to form a further-magnified image of a section of the first image on a fluorescent screen (7) for visual observation, or on the sensitive emulsion of a photographic plate (8). The magnitude of this further electron magnification is adjusted so that the finest details reproduced in the final image have dimensions at least one order of magnitude greater than the grain size of the photographic emulsion. As the maximum area of exposed photographic emulsion is a constant for any particular instrument, it is desirable to use emulsions with the finest possible grain size so that the total electronic magnification can be kept at a minimum and the corresponding field of view at a maximum. Moreover, as the speed of a photographic emulsion is very roughly proportional to the grain size, while the intensity in the electron image varies inversely as the square of the magnification, the finer grain sizes with the corresponding lower electronic magnification give shorter exposure times. Thus by the use of fine-grain emulsions, large fields of view can be recorded on small areas of photographic emulsion, and may be subsequently enlarged by optical means to the total magnification necessary to bring all details to a size where they may be observed by the unaided eye.

Resolving Power

According to de Broglie's work, the wavelength λ, which is to be associated with electrons traveling at the velocity imparted by a potential drop of V volts, may be expressed by the formula

$$\lambda = \sqrt{\frac{150}{V}} \times 10^{-7} \text{ mm.} \qquad (2)$$

From this the wavelength associated with a beam of electrons accelerated by 60,000 volts is found to be only 0.05×10^{-7} millimeters. The wavelength of light in the visible spectrum, on the other hand, is of the order of 5000×10^{-7} millimeters. Although, by substitution of the value of λ for 60-kilovolt electrons in (1), we can see that from the diffraction theory it should be possible to resolve details with dimensions even smaller than atoms; electron lenses, in their present state of development, have defects of a magnitude which prevents even an approach to the theoretical maximum attainable resolving power.

The magnitude of the effect of the lens defects on the quality of the image depends on the third and fifth powers of the divergence from the lens axis of the paths of the image-forming rays leaving the specimen and entering the lens, while the corresponding effect of the diffraction defect is merely inversely proportional to the same divergence. Hence, by restricting this divergence by the use of a small limiting aperture, the effect of the aberrations of an electron objective lens

can be decreased many orders of magnitude with only a relatively small increase in the magnitude of the diffraction defect. With present electron objective lenses, an optimum value of aperture, i.e., that for which the magnitudes of the effects of lens aberrations

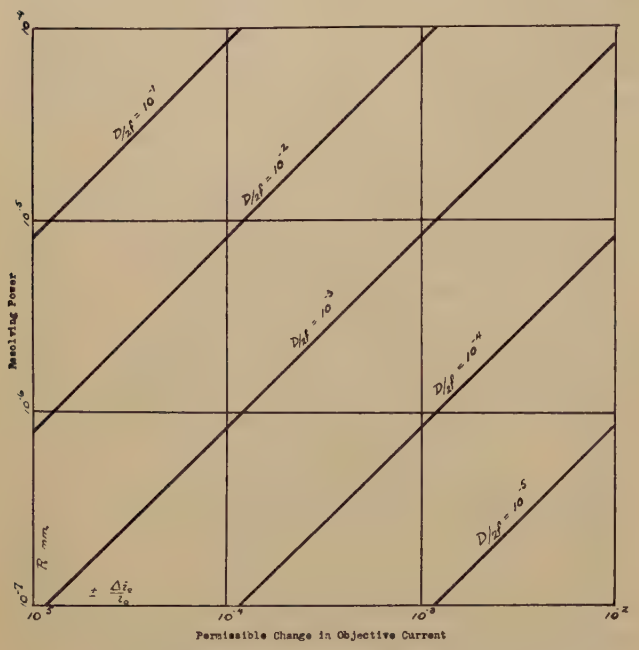

Fig. 3—Relation between resolving power and fluctuation in objective-lens coil.

and diffraction defect are equal for the aperture ratio, is approximately

$$\frac{D}{2f} = 2 \times 10^{-3} \text{ (for 60 kv)} \qquad (3)$$

where D = diameter of the aperture opening, and
f = focal length.

The expression $D/2f$ corresponds to the numerical aperture $\mu \sin \alpha$ of the light microscope and can be substituted in (1) to obtain a rough estimate of the magnitude of the diffraction defect.

$$d = \frac{0.61\lambda}{\frac{D}{2f}} = 15 \times 10^{-7} \text{ mm.} \qquad (4)$$

In actual practice, the size of the physical aperture used varies from this optimum value to take into account the contrast and resolution desired with different types of specimens.

Owing to the fact that the strength of the magnetic electron lens depends upon the value of the current flowing through the exciting coil and on the velocity of the electron beam, it is obvious that if photographic records which contain all the information present in the electron image are to be obtained, it is necessary that the exciting current and the beam velocity remain extremely steady during the time of exposure. For a given magnetic objective, the degree of steadiness required in the exciting current and the velocity of the electron beam can be very easily calculated by consideration of the size of the circle of confusion produced in the image by known variations in the electrical supplies. The relation between the desired resolving power and the maximum permissible fluctuation in the objective-lens-coil current for different apertures is shown in Fig. 3. The curves given are for only those image points which lie on, or extremely close to, the optic axis of the objective, and hence they must be modified slightly if all points of the image are to be considered. This modification must take into account the change in magnification and rotation of the image which always accompanies changes in the power of the lens. Fig. 4 shows a similar set of curves which give the relationship between the desired resolving power and the maximum permissible variations in the accelerating potential for different lens apertures. This set of curves can also be used to determine the effect on the resolving power of a velocity distribution introduced in the beam of electrons by the specimen. Both sets of curves have been calculated for an objective magnification of 100 times.

In addition to the disturbing effects of the diffraction defect and of the instabilities in the electrical supply, there are a number of others which must be eliminated. These include the effects of varying external magnetic fields, electrostatic potentials which may build up on partially insulating surfaces on lens diaphragms in the path of the electron bombardment, and mechanical

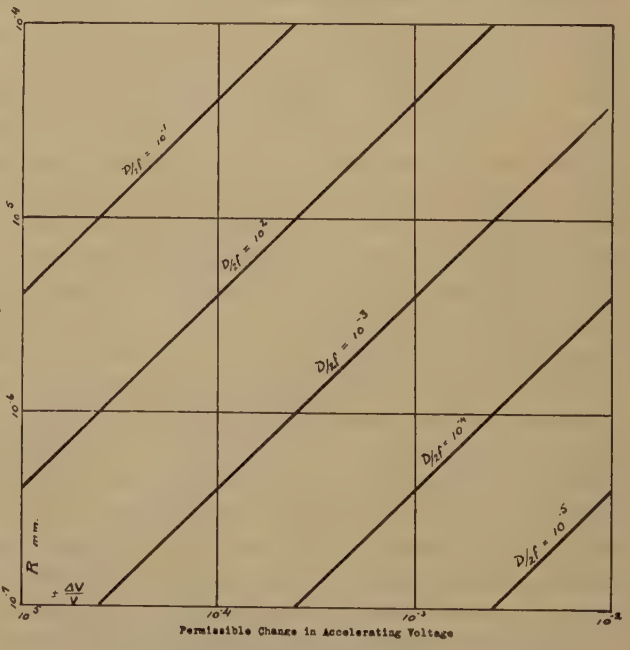

Fig. 4—Relation between resolving power and variations in accelerating potential.

vibration which causes independent displacement of any of the components of the optical system during exposure. The effects of mechanical vibration and of varying external magnetic fields can be eliminated quite easily by rigid construction and proper shielding of the sensitive regions. The effects of electrostatic

potentials on the objective diaphragm present a more serious difficulty. It has been found that at the pressures obtained in a demountable system, a somewhat carbonaceous semiconducting material is deposited on all the diaphragms which are in the direct path of the electron beam. When these deposits become sufficiently heavy, the potentials of those parts of their surfaces which are exposed to the electron beam may become considerably different from the potential of the metal aperture. The effect is most serious in the case of the limiting aperture of the objective lens, where the action is such that a very poor electrostatic lens is formed in the plane of the limiting-aperture opening. The image defects caused by this electrostatic effect have been found sufficient to limit seriously the ultimate resolution attainable with the magnetic objective lens. With the present objective, for instance, a resolving power of better than 30×10^{-7} millimeter is possible with a new aperture, but as it is used and the deposits are formed, the resolving power gradually grows poorer until at the end of about two months of normal operation a resolving power of better than 100×10^{-7} millimeter is no longer attainable. At present the simplest method of avoiding this difficulty seems to be the periodic replacement of the limiting aperture.

The New RCA Electron Microscope

Previous electron microscopes which have been able to attain a resolving power of 100×10^{-7} millimeter or better have had common disadvantages in the large amount of space occupied by the high-voltage electrical equipment, special installation requirements, difficulty of operation, and the low percentage of photographic records showing the highest attainable resolution. In an effort to eliminate these disadvantages, the present electron microscope has been designed with compactness, reliability, and simplicity of operation as essential requirements, insofar as they do not limit the ultimate attainable resolution.

A schematic cross section of the complete unit is shown in Fig. 5 and a photograph of it in Fig. 6. The development of the high-frequency high-voltage supply has made possible the incorporation of all the electrical equipment, the microscope proper *A* to *E*, and the high-speed oil-diffusion pump *H* into one structural unit. There are a number of advantages in such an arrangement: For example, the high-tension connection to the microscope is very short and is completely enclosed in a grounded, lead-lined case. Moreover, the control panel, which is within comfortable reach of the operator in position for viewing the final image, is situated close to the equipment to be controlled so that long complicated connecting cables are eliminated.

The microscope proper is built up of a number of separate units which are fastened together by means of specially constructed mechanically rigid and vacuum-tight junctions. This sectional construction not only allows the easy replacement of separate parts in the case of failure but also enables new developments to be incorporated in existing equipment. Synthetic-

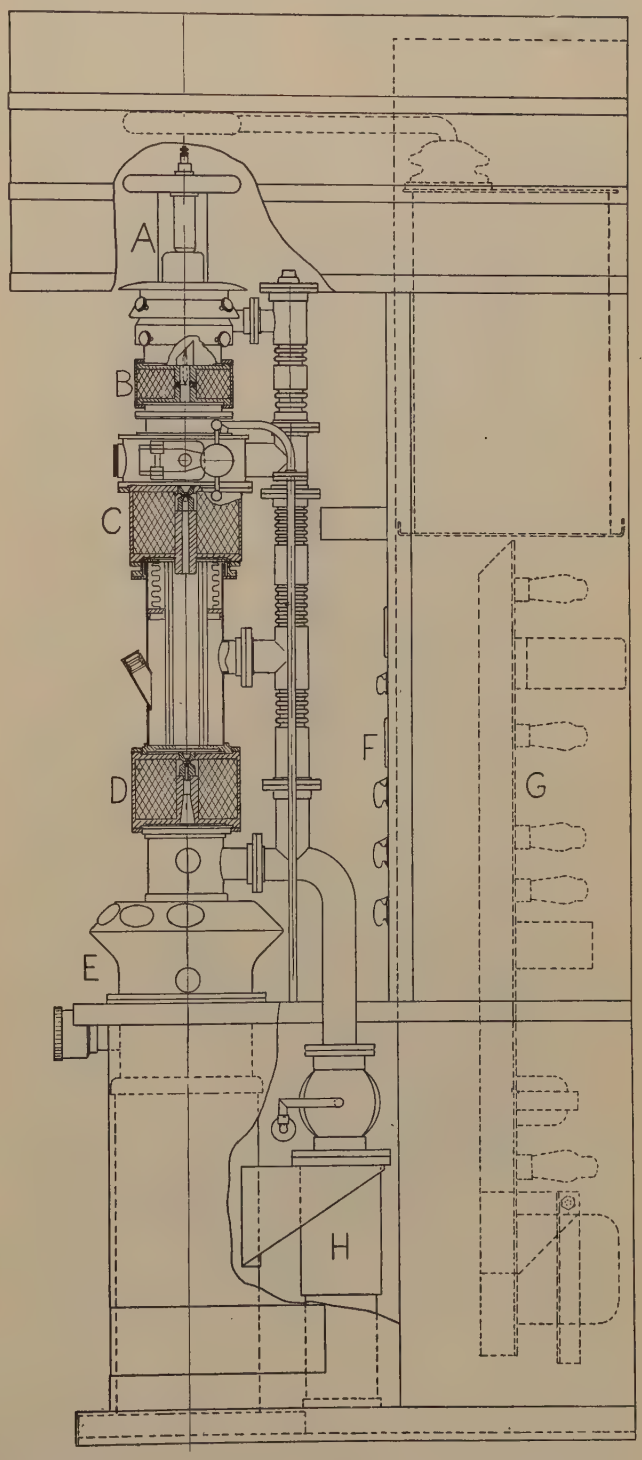

Fig. 5—Schematic cross section of complete electron microscope.

rubber gaskets are used at all the demountable vacuum seals, while flexible metal bellows are used wherever motion must be transmitted to parts in the vacuum chamber. The vacuum seal between adjustable sections of the microscope is also maintained by flexible metal bellows.

The electron source *A*, which forms the top unit of the microscope, can be adjusted by two sets of adjusting screws so that the electron beam leaving the source can be made to coincide with the axis of the condenser. The upper surface of the condenser lens *B* carries a fluorescent screen on which can be observed a cross section of the electron beam. An opening located in the center of this screen allows that part of the beam utilized in the irradiation of the specimen to pass into the condenser lens. A set of adjustment screws immediately below the condenser allows it and the electron source to be adjusted as a unit in a plane perpendicular to the axis of the microscope. By this means the axis of the whole illuminating system can be made to coincide with the axis of the objective. Immediately below the condenser is the object chamber which contains the movable object stage and the object-changing mechanism. The latter consists of an air-lock system and the associated vacuum valves, so arranged that the specimen can be replaced quickly and simply, without breaking the vacuum of the main chamber. The manipulation of one handle operates a lever system which lifts the specimen out of the object stage and places it in the air-lock chamber at the same time as it closes the inside opening to that chamber, thus isolating it from the main vacuum chamber. The operation of unlatching and opening the outer door to the air-lock chamber automatically operates the valves which break the vacuum in that chamber. The specimen can then be removed from the air-lock chamber and replaced by a new one, and the reverse procedure followed.

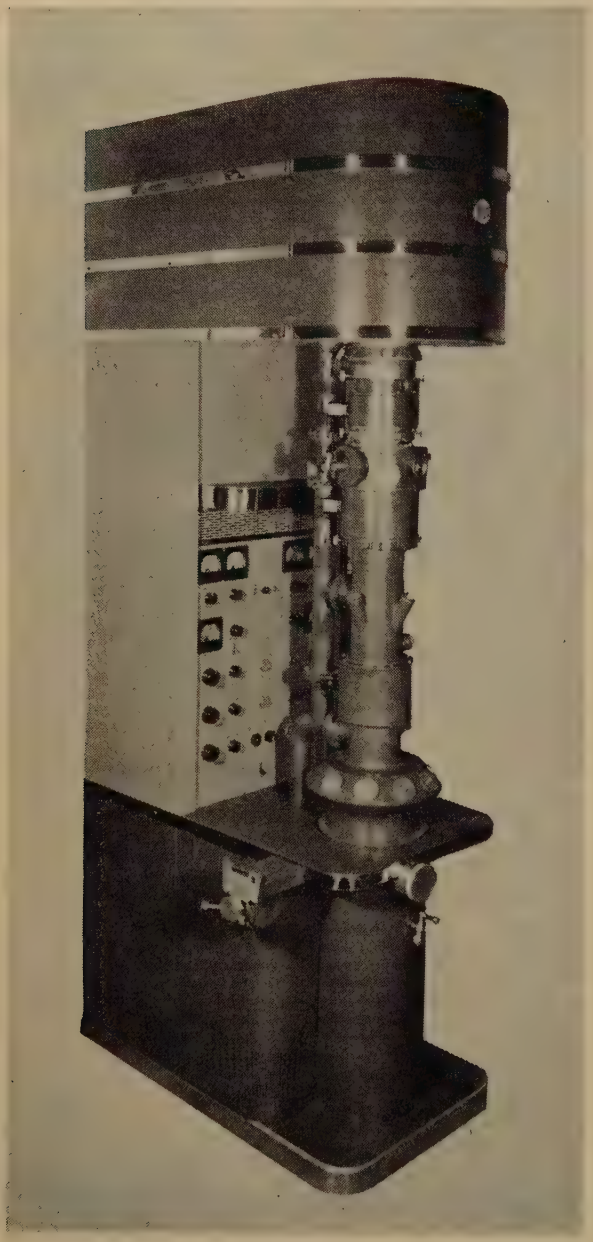

Fig. 6—Photograph of microscope.

The objective lens *C* consists of a coil which, with the exception of a gap located at the end of the central opening, is completely encased in iron. The walls of this gap form a precisely machined receptacle for the pole-piece unit. The necessity for water cooling has been eliminated by the design of a coil which requires an input of less than 5 watts. The pole-piece system of this objective and the limiting aperture with its support and centering adjustments form a unitary assembly. A special procedure developed for the machining of the pole pieces and the nonmagnetic spacer enables the axes of the pole pieces to be maintained in exact coincidence. To align the axis of the objective and the illuminating system with the axis of the projector, an adjustment is provided immediately below the objective.

The image produced by the objective can be viewed on a fluorescent screen mounted on the upper surface of the projection lens *D*. The latter is similar to the objective in every respect except the size of the pole-piece opening and limiting aperture. The highly magnified final image produced by this lens is projected on a large fluorescent screen on the bottom plate of the viewing chamber *E*. The latter is provided with six viewing ports so that a large group can view the image simultaneously. The ports are placed so that binocular observation of the final image is possible. Immediately below the viewing chamber is situated the photographic plate which contains a 10-×2-inch plate. As in the case of the specimen, the plate can be changed through an air lock without breaking the vacuum in the main chamber. The use of a long plate enables a number of exposures to be made at one loading; the size and number of exposures per plate can be varied according to the requirements of the work in hand. The fluorescent screen, which is hinged and connected to an external knob, is used as a shutter.

The microscope is mounted on a large iron pedestal with a flanged base, which is in turn mounted on the base plate of the whole unit. All mechanical connections between the microscope and the remainder of the unit, and between the unit and the floor, are made

through rubber pads to minimize the effects of building vibrations.

The vacuum system consists of a separate mechanical fore pump and a fast oil-diffusion pump. The latter is mounted on the pedestal and pumps the different sections of the microscope through a large manifold. A second mechanical pump, which is also mounted separately from the microscope unit, is used to exhaust the air-lock chambers. A pressure of about 10^{-5} millimeter is maintained in the main chamber.

The Electrical Supplies

In an earlier paper[9] one of the writers has already described a type of power supply for electron microscopes. For the present microscope, however, it was necessary to redesign the power supply completely in order to obtain the reduction in size that was desired. The resulting compactness has been achieved chiefly by redesigning the high-voltage supply and its regulator system. The main feature of this new design is the actuation of the high-voltage rectifiers by radio frequency in place of 60-cycle power. Actuation by radio frequency as compared with 60-cycle power has a number of advantages in the production of extremely steady high voltages where the current drain is low. Some of these advantages are as follows:

1. Much smaller condensers are required for given ripple output.

2. Low-loss resonant coils requiring very little exciting power can be achieved at these frequencies in units which occupy only a small space and are light in weight. As the maximum load of 0.5 milliampere at 60,000 volts represents only a small part of the total power, the efficiency of the resonant circuits is a matter of considerable importance.

3. The voltage regulator operating on the low-voltage input side of the rectifier is limited in speed by the frequency of the supply. Radio-frequency operation reduces this effect to negligible proportions.

4. While it is exceedingly difficult to shield the microscope from the strong fields of a closely situated 60-cycle high-voltage transformer it is very easy in the case of radio-frequency high-voltage coils.

A block diagram of the high-voltage unit is given in Fig. 7. The high-voltage coil L_0 is designed to resonate with the total effective capacitance C_0 at the frequency at which the maximum impedance is obtained. This determines the operating frequency, which is 32 kilocycles in this case. The whole rectifier circuit, including L_0, is immersed in an oil-filled tank. The rectifier presents a series-resonant radio-frequency load to the driver oscillator. A special two-stage oscillator circuit, whose oscillating frequency is determined by the tuned load, was developed. The oscillator output may be modulated over a wide range by varying the screen voltage input to the power stage. The main principles of the feedback regulator are the same as have been described.[9] The high-voltage output is divided by a special potentiometer. (This potentiometer is included in the direct-current modulating amplifier block in Fig. 7.) The divided voltage is compared to a standard battery, and the difference impressed on the input to a direct-current modulating amplifier which drives the screens of the output stage of the driver oscillator. A very stable feedback regulator is thereby obtained, which maintains the output voltage constant to 0.004 per cent, or better, over any 30-second exposure period. This stability was measured by means of a voltage divider of known stability and a standard battery, as was described in the earlier paper. As shown in Fig. 7, the filament of the microscope is supplied by

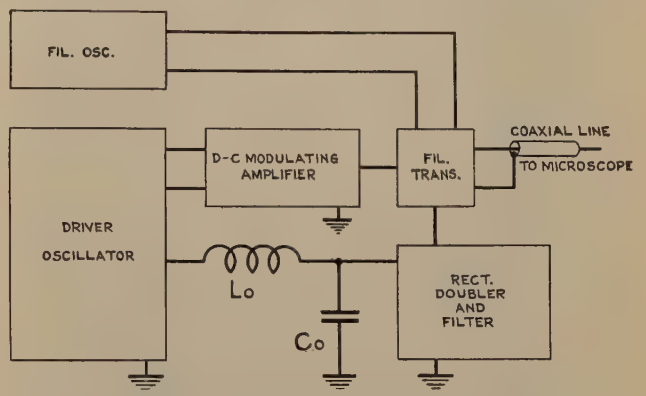

Fig. 7—Block diagram of power supplies.

a radio-frequency oscillator through a step-down transformer. The input direct voltage to this oscillator is obtained from a conventional electronically regulated supply, in order to minimize changes in emission with line-voltage variations. The total current load on the high-voltage supply is read on the ground side of the rectifier on an 0.5-milliampere meter. Included in series with the meter is an overload relay, which closes on sustained overloads. The relay turns on a warning light and renders the driver oscillator inoperative by applying a large negative grid bias. Once overloaded, the relay holds in until released by a push-button switch. A special circuit is provided to protect the meter and relay from surges accompanying sudden short circuits and overloads.

The regulated current supplies for the magnetic lenses of the electron microscope are essentially the same as those described before.[9] The currents required are smaller and the coil resistances are less than required for the earlier 100,000-volt microscope. The greatest stability requirement is for the objective coil, which requires constancy to 0.0074 per cent. All input voltages—filament, plate, and screen—to the objective-coil regulator are obtained from regulated sources.

The objective current, therefore, is doubly regulated against line-voltage changes and the stability is further insured by minimizing drifting in the amplifier, so that this unit exhibits short-time stability better than 0.002

[9] A. W. Vance, "Stable power supplies for electron microscopes," *RCA Rev.*, vol. 5, pp. 293-300; January, 1941.

per cent as well as excellent long-time stability. The projection- and condenser-coil regulators are of the conventional type.

The complete electrical system consists of three units: the control panel, the chassis, and the high-voltage oil tank. The control panel contains the main power switch, high-voltage switch, high-voltage control switch, and controls for condenser-, projection-, and objective-coil currents, all within ready reach of the microscope operator when seated in the position for viewing the magnified image on the fluorescent screen. The condenser-coil current is controlled over a 6-to-1 range on a single smooth linear control. Linear steps of electron magnification are obtained by controlling the projection-coil current, over a 10-to-1 range in 21 square-root-law steps. The objective-coil current control covers a range of 7 to 1 and can be adjusted to within 0.04 per cent anywhere in this range by means of two decade switches and a smooth potentiometer. The final focusing of the electron image is accomplished with the objective-coil-current control. By means of a multicontact switch and appropriate multipliers and shunts, a single meter is used to read the three coil currents as well as the various oscillator plate currents and the output voltages of the different regulated supplies. The high voltage is not metered but is read off directly on the high-voltage-control switch which indicates 5-kilovolt steps. Since the accuracy of con-

Fig. 8—Photograph of rear of microscope showing electrical supplies.

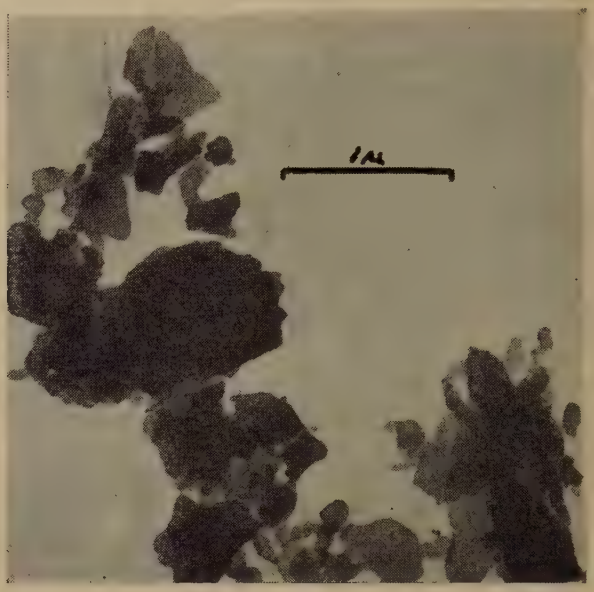

Fig. 9—A finely ground form of a lead arsenate insecticide, showing characteristic platelike structures. Magnification, 22,000.

Fig. 10—Aerobacteria cloacae, showing internal structure of organism and fine flagella. Magnification, 18,000.

trol is much greater than meter accuracy, projection- and objective-coil current data are obtained from the control-switch indications rather than from the meter. Pilot lights are provided for main power, high voltage, and high-voltage overload. In the event of overload, the overload-release button may be pressed, and if the fault has cleared, the high voltage will return to normal and the overload pilot will go out, but if the overload still exists, the high voltage will not come on and the pilot will stay lighted. Conventional vacuum-measuring equipment, provided with completely independent power supplies which may be operated when the main power switch is off, is placed at the top of the panel.

The complete power supply is contained in a standard cabinet rack, 7 feet high, $20\frac{1}{2}$ inches wide, and $14\frac{1}{2}$ inches deep. The hood enclosing the upper part of the microscope fastens directly to the top of this rack and contains the high-voltage connection between the rectifier and the microscope. The connection lead consists of a polished 1-inch tube with corona shield over the microscope cathode; through this tube passes a flexible lead carrying one side of the electron-microscope-filament current, the tube itself completing the circuit. The tube and corona shield are supported directly on the high-voltage bushing on the oil tank, which is mounted in the top of the cabinet rack. The only physical connection to the microscope is through two flexible leads from the corona shield to the micro-

Fig. 11—Bovine tubercle bacillus, showing internal structure and the fine material on background. Magnification, 28,000.

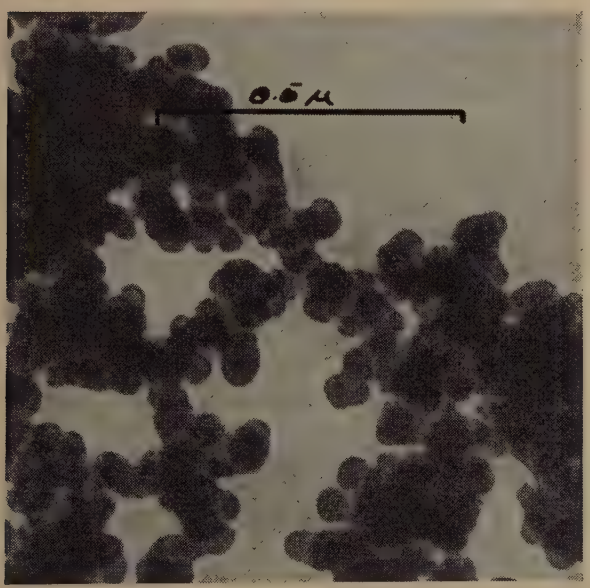

Fig. 12—Carbon-black particles deposited from camphor flame. Magnification, 80,000.

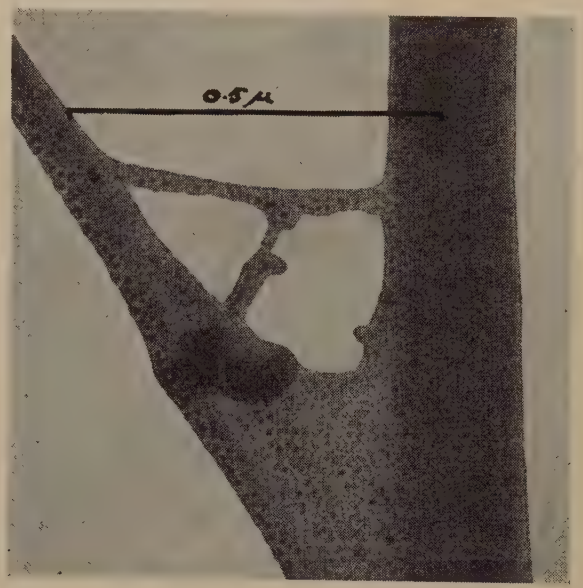

Fig. 13—A polymer of vinylchloride in the form of thin filaments. Magnification, 100,000.

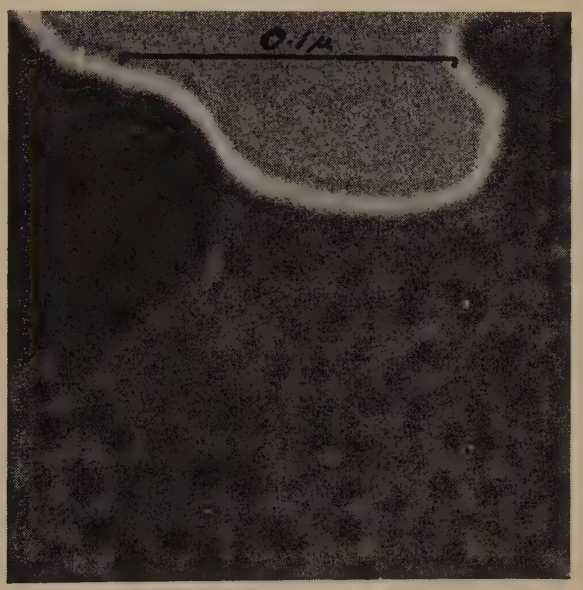

Fig. 14—A section of Fig. 13 enlarged to 450,000 to show the fineness of detail present.

scope cathode, this arrangement preventing mechanical vibration from the rack reaching the microscope.

Fig. 8 shows the back view of the rack with the door open. A fan is provided in the door to promote air circulation. All incoming connections to the high-voltage oil tank, which is shown at the top, are made through coaxial plugs and cables which are also plugged into the chassis. The chassis is hinged so that it may be laid back horizontally for servicing when the coaxial plugs leading to the oil tank are pulled out. The unit is completely interlocked so that the power is off when either the upper hood or back door is opened.

The supply system just described has been in use for several months in connection with the new RCA electron microscopes and the results have been extremely gratifying, measured stabilities being well in excess of those required. In no exposures obtained to the time of writing (over 2000) has the resolving power been limited by changes in the power supplies. Excellent repeatability of adjustments is shown by the fact that exposures have been made with good resolution merely by setting the controls to the values used for previous exposures and without observing the electron image or doing any focusing. The high-voltage supply has negligible regulation, making it possible to vary the image intensity widely without movement or change of focus of the image. The value of the overload-protection circuit has been demonstrated in several instances when the vacuum pumps were accidentally shut off and the ensuing gas discharge overloaded the unit and closed the overload relay. When the pumps were started, the vacuum regained, and the overload relay button pressed, the image came back on the screen exactly in focus. With the brute-force stabilizing systems usually used with electron microscopes, such a discharge would seriously damage or burn out the filament and the instrument would probably have to be dismantled and cleaned out before it could be used again.

Electron-Microscope Images

As mentioned above, the intensity distribution in the electron image formed by the electron microscope is, for the most part, due to the different scattering powers of the different points of the specimen. The use of a very small limiting aperture in the objective allows only those electrons which have been deflected through an angle less than a certain maximum angle, determined by the size of the limiting aperture, to reach the image. In general, absorption of the electrons does not occur in the extremely thin type of specimens best suited for electron-microscope observation and, in fact, is to be avoided as the energy given up by the absorbed electrons appears as heat and may be sufficient to raise the temperature of the specimen to the point where decomposition and structural changes can occur. It has been found in practice that with electrons accelerated by less than 100,000 volts, specimens which are more than 10^{-3} millimeter thick do not give satisfactory images. While this imposes a limitation on the range of application of present electron microscopes, there are, nevertheless, innumerable fields of investigation where the electron microscope is of immediate use. These include the studies of bacteria, viruses, large molecules, clays, cements, pigments, ores, and colloidal particles of all kinds. A few examples are shown in Figs. 9 to 14. The photographs reproduced serve to illustrate the high resolution obtainable with the present instrument. They represent a small selection from some 1000 exposures made in the course of the testing of the present instrument during July, August, and September of 1940. During this test it was found that under routine working conditions the resolving power of the microscope was somewhat better than 100×10^{-7} millimeter in about 75 per cent of the exposures. However, with a clean objective aperture, extremely precise alignment, and with a suitable specimen, the instrument has been found to have a resolving power of better than 30×10^{-7} millimeter.

Acknowledgments

We wish to express our appreciation of the many useful suggestions and criticism given by Dr. V. K. Zworykin, under whose supervision the present electron microscope was designed and constructed. We are also greatly indebted to Mr. J. M. Morgan for his valuable assistance in the design and test of electrical equipment.

A Stabilized Frequency Divider*

GEOFFREY BUILDER†, ASSOCIATE, I.R.E.

Summary—*A method is described for stabilizing the frequency of a relaxation oscillator to any required degree. The stabilized oscillator may be synchronized with a high-order submultiple of a control frequency without any possibility of synchronization at any other submultiple, even if the operating conditions vary widely; the maintenance of synchronization can also be made substantially independent of operating conditions.*

I. Introduction

FREQUENCY division now finds extensive use in systems in which it is desired to generate oscillations at frequencies rigorously determined by a standard or substandard of higher frequency. Among the best known of these applications are those in frequency-measuring equipment[1–6] and those in the generation of carrier frequencies for wire-line carrier systems.[7]

Three methods of frequency division by electrical means may conveniently be distinguished; namely, (a) synchronization of a relaxation oscillator at a submultiple of the control frequency; (b) synchronization of a regenerative oscillator at a submultiple of the control frequency; and (c) excitation by the control frequency of an active network which is tuned to the required subharmonic frequency but which gives zero output when the control energy is zero. Of these, the relaxation and regenerative oscillators may be regarded as extreme cases of a general type of oscillator.[8,9] The relaxation oscillator is characterized by an extremely distorted wave form, an inherent lack of frequency stability, and a high degree of susceptibility to synchronization even at very high-order submultiples of a control frequency. The regenerative oscillator is characterized by a close approach to a sinoidal wave form, a relatively high degree of frequency stability, and a correspondingly smaller susceptibility to synchronization. The use of the driven type of frequency divider[7,10,11] is analogous to the process of frequency multiplication by harmonic distortion and the selection of the required harmonic by filtering, it has obvious advantages but does not seem to have yet been developed to the stage of practical usefulness achieved by synchronized arrangements, at least for frequency division by large factors.

Synchronization of a simple relaxation oscillator as a means of frequency division is readily achieved but has serious practical disadvantages, the chief of which is the close control of the operating conditions required to prevent failure of synchronization or a change of synchronization to a submultiple of the control frequency other than that required. We have analyzed these conditions of synchronization in some detail in a recent paper[12] and they have also been discussed by others.[13,14] To overcome these difficulties the use of regenerative oscillators inherently incapable of oscillation except at frequencies near that of the required subharmonic has been discussed from time to time[15,16] but little practical application has resulted; the susceptibility to synchronization is apparently dependent on the existence of a poor wave form[17,18] and is difficult to control. A compromise was achieved by Pulley and Saxby,[4] but their circuit is not easily analyzed and hence somewhat difficult to design satisfactorily in practice.

Byard[19] has made the suggestion, based on the work of Eccles and Leyshon,[20] that a system of selective feedback, through a circuit tuned to the required subharmonic, could be associated with a relaxation oscillator to favor the generation of the required subharmonic of the control frequency. Earlier we had developed such a system for frequency measurement[5] and have found that it is readily analyzed and designed, and that it gives an extremely satisfactory performance.

The present paper discusses in some detail the frequency stabilization of a simple relaxation oscillator by selective feedback and the synchronization of the stabilized oscillator as a frequency divider at high

* Decimal classification: R355.6×R213. Original manuscript received by the Institute, October 1, 1940.

† Amalgamated Wireless (Australasia), Ltd., Ashfield, N. S. W., Australia.

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orders of division. The method of stabilization is applicable to any type of relaxation oscillator and may be useful in stabilizing the frequency of self-excited power inverters. The investigation has been confined to the use of a gas-triode relaxation oscillator, but the results should be qualitatively applicable to any other type.

II. Stabilization of a Relaxation Oscillator

The basic principle of the method developed is indicated by the block-schematic diagram of Fig. 1. The

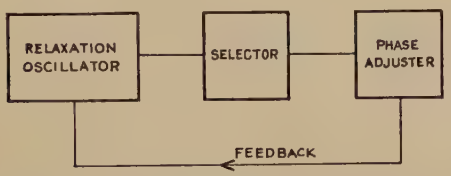

Fig. 1—Basic arrangement for the frequency stabilization of a relaxation oscillator.

output from any relaxation oscillator is applied to a device selectively responsive at the frequency required; in general, the degree of selectivity determines the degree of stabilization achieved. The output from the selector is fed back to any element in the relaxation oscillator capable of changing the relaxation frequency, and the phase and amplitude of the feedback energy is chosen so that oscillation occurs in the vicinity of the selector frequency, or not at all.

Circuit Arrangement

The practical application of this principle may lead to a variety of designs. That of Fig. 2 is chosen for illustration because in it the functions of relaxation, selection, and feedback are substantially independent and hence more readily analyzed; a description is given later of a circuit that is more economical in components. In Fig. 2, the functions of the main sections

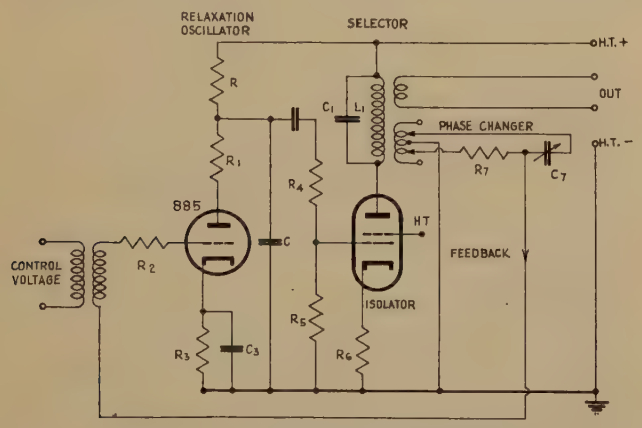

Fig. 2—A circuit arrangement in which the functions indicated in Fig. 1 are substantially independent.

of the circuit are indicated, and there is shown an input transformer through which a control voltage may be applied for synchronization of the stabilized oscillator.

The *relaxation oscillator* in Fig. 2 is of the simple type using a gas-filled triode type 885, and its natural relaxation frequency is determined by the time constant RC, by the high-tension voltage, and by the cathode bias developed across the parallel combination R_3C_3; the resistance R_1 has a low value and prevents excessive instantaneous discharge currents through the valve, while R_4 has a high value (e.g., 0.25 megohm) to prevent the grid-cathode voltage attaining a voltage substantially greater than zero however large the voltages applied to the grid circuit may be. The *selector* is a circuit L_1C_1 tuned to the frequency to be generated, and is fed from the relaxation oscillator through the potential divider R_4, R_5 and a pentode-valve isolating amplifier that is biased by the cathode resistor R_6. Secondary windings on the inductance L_1 provide output and feedback voltages. Taps on the feedback winding permit a choice of the amplitude of the feedback voltage; the center tap of the winding is earthed and across it is connected in series the resistance R_7 and the variable capacitance C_7; the feedback voltage is taken between the junction point of

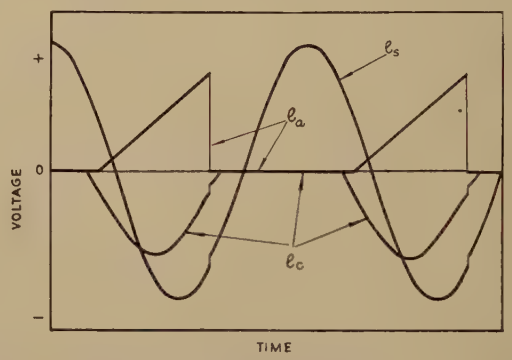

Fig. 3—Wave forms of voltages generated in an oscillator using the circuit of Fig. 2.

C_7R_7 and earth. The center-tapped transformer secondary in combination with R_7C_7 form the well-known constant-voltage *phase changer* described by Hartshorn in 1937 and earlier authors; it gives phase changes up to $\pm\pi/2$, and larger changes can be obtained by reversing the connections to the transformer secondary.

Analysis of Operation

The mode of operation of the circuit is indicated by Fig. 3, in which curves are plotted, against time, of the voltages at various points in the circuit with respect to earth. The voltage e_s across the tuned selector circuit is substantially a sine wave of the required frequency, and it will be assumed that the selector L_1C_1 has zero power factor at this frequency. The voltage e_s is fed back to the grid circuit of the relaxation-oscillator valve after passing through the phase-changing network. The voltage e_c actually applied between grid and cathode of the relaxation-oscillator valve is also shown, and is made up of the bias E_c provided by the cathode-bias circuit R_3C_3 together with the feedback voltage; but the combined voltage is prevented from rising appreciably above zero by the high resistance R_2 in series with the grid.

The operation of the 885 gas-filled triode is well known. Anode current flows only when the anode voltage exceeds a critical value determined by the grid-cathode voltage; this critical striking voltage is roughly equal to ten times the negative grid voltage. Anode current ceases when the anode voltage drops below a critical extinction voltage which is constant at about 10 volts for negative values of grid voltage; for simplicity it will be assumed that the critical extinction voltage is zero, as this does not materially affect a general discussion of the circuit.

Fig. 3 shows that while the grid voltage e_c remains about zero, i.e., during the greater part of the positive half cycle of the feedback voltage, the 885 triode is conducting and the anode remains at about zero voltage. As soon as the grid voltage e_c becomes negative, anode current in the triode ceases and the critical striking voltage rises rapidly to high values; the condenser C then starts to charge up, and this continues until the grid potential again rises towards zero, when the anode striking potential drops to the voltage then attained at the valve anode, i.e., across the condenser C. It will be observed that the instant of striking is very precisely determined by the simultaneous rise of anode and grid voltages. The resulting wave form at the anode of the 885 valve is shown in the curve e_a of Fig. 3, and it will be observed that it is of the familiar "saw-tooth" form, alternating with periods of about equal length during which the voltage is zero. It has been assumed that the rate of increase of anode potential during each charging period is constant, and this is approximated in practice if the high-tension supply voltage is several times the anode striking potential.

The curve e_a also gives the form and phase of the potential applied to the grid of the isolator valve. It is apparent that for equilibrium the fundamental component of the wave e_a must be in exact antiphase with the voltage e_s developed across the selector circuit. If the circuit is to oscillate at the selector frequency the phase must be adjusted in the feedback path, by means of the phase shifter provided, to fulfill this condition.

Design Considerations

The following tabulation sets out briefly the main considerations in design and at the same time indicates somewhat more closely the operation of the circuit.

(a) The natural relaxation frequency can conveniently be equal to, or somewhat less than, the required oscillation frequency; the choice is in no way critical and has little effect other than altering the rate of charge of the condenser and hence the voltage attained in charging during each cycle. If the feedback voltage is not made very great or if the natural relaxation frequency is made very high two or more relaxations may occur in each period; this is not serious but changes the wave form and requires readjustment of the feedback phase.

(b) The feedback voltage should be made great enough to achieve the type of operation indicated by the curves of Fig. 3, e.g., a root-mean-square value of feedback voltage of 20 volts is suitable for a device operating on a high-tension supply voltage of 250 volts and with relaxation amplitudes up to 100 volts. The use of a large alternating voltage for rigid control of phase and frequency is well known.[21]

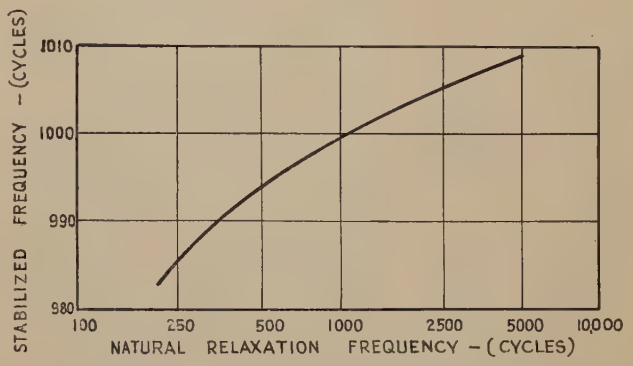

Fig. 4—The variation in stabilized frequency, for an oscillator using the circuit of Fig. 2, with variation in the natural relaxation frequency. The selector circuit had a magnification factor (Q) of 6.

(c) The phase-shifter should be adjusted so that the actual oscillation frequency is that of the selector circuit L_1C_1. This is the usual condition for stabilization[22] and ensures that, for any change of operating conditions, the change of frequency needed to readjust the phases is a minimum and is the same for phase divergences in either direction.

(d) The selectivity of the circuit L_1C_1 should be chosen in accordance with the degree of stabilization required.

Performance

The degree of stabilization that can be readily achieved is illustrated by Fig. 4, taken for the circuit of Fig. 2 adjusted as outlined above, and using a selector circuit L_1C_1 having an effective magnification factor $Q=6$. The unstabilized natural frequency of the relaxation oscillator was varied from 200 to 5000 cycles by varying the cathode resistor R_3; the corresponding total change in the stabilized frequency amounted to only 3 per cent.

III. Frequency Division

The foregoing method is admirably suited to the stabilization of a relaxation oscillator to be used for high-order frequency division. The stabilized oscillator differs from a regenerative oscillator in that its output contains a much greater proportion of energy at high harmonics of the generated frequency, even if the lower-order harmonics are no greater. This is indicated by Fig. 3, in which the output wave is closely sinoidal in general shape but has a small discontinuity, caused

[21] G. Builder, *Jour. I.E.E.* (London), vol. 73, p. 419; 1933.
[22] F. B. Llewellyn, "Constant frequency oscillators," Proc. I.R.E., vol. 19, pp. 2063–2094; December, 1931.

by the abrupt change in voltage when the relaxation oscillator attains its critical potential. The richness in high-order harmonic content is conducive to synchronization by energy at the high-order harmonic frequencies.[17,18] On this view, the susceptibility to synchronization by energy of any one harmonic frequency depends on the sharpness of the stabilizing selector circuit and therefore, as is to be expected, on the degree to which the relaxation oscillator is stabilized. The selector-circuit characteristics, therefore, must be chosen appropriately to the application; e.g., for frequency division, the degree of stabilization should be great enough to prevent accidental synchronization at submultiples adjacent to that required, but should be low enough for synchronization to be maintained over a sufficient range of differences between the stabilized oscillator frequency and the required synchronized frequency.

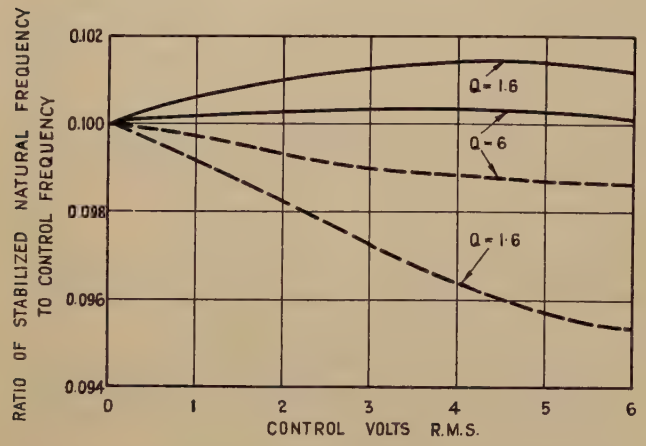

Fig. 5—The range of synchronization of an oscillator using the circuit of Fig. 2 with a feedback of 15 volts root-mean-square from the selector.

A control voltage may be applied to the stabilized oscillator of Fig. 2 by means of the input transformer shown, the transformer secondary being connected in series with the grid circuit so that the control voltage is added to the stabilizing feedback voltage. The range of control frequencies over which synchronization is maintained can readily be determined for any particular set of conditions. Fig. 5 shows the range of synchronization, in terms of the ratio of the stabilized natural frequency to the control frequency, for frequency division by a factor of 10 to 1, using selector circuits with circuit magnification factors (Q) of 6 and 1.6. Upper limits of this ratio are indicated by full-line and lower limits by broken-line curves. The voltage fed back to the grid circuit of the oscillator for stabilization was approximately 15 volts root-mean-square. The curves show the variation of synchronization range with an increase of control voltage, and it is to be observed that the range increases with increase of control voltage up to about 5 volts root-mean-square. The more limited range of synchronization with the higher selectivity circuit is marked, but a range of at least 6 per cent is attained with the circuit of lower selectivity. Thus the range of synchronization is readily controlled and a wide range is readily achieved if it is required.

The shape of the curves shown in Fig. 5 is noteworthy. The range of synchronization is very much less for natural stabilized frequencies greater than one tenth the control frequency than it is for lower natural frequencies. With further increase in the control voltage, synchronization may, in some cases, occur only if the natural stabilized frequency is less than one tenth of the control frequency. These effects were well established by the experimental work carried out. Although this asymmetry has not been successfully analyzed in detail for the present case, it has been found that very similar results are obtained in the synchronization of a simple relaxation oscillator. The latter case has been analyzed in detail, the results being published in a recent paper,[12] and there seems no doubt that the mechanism of the effect is substantially the same in the present more complex case. Fig. 6, from the paper by Builder and Roberts,[12] shows for a simple gas-triode relaxation oscillator, by the shaded areas, the ranges of the ratio of natural relaxation frequency f_0 to control frequency f over which synchronization occurs. These curves were calculated on the assumption that the critical extinction voltage of the gas triode was a constant, or zero, but were confirmed experimentally after being corrected in conformity

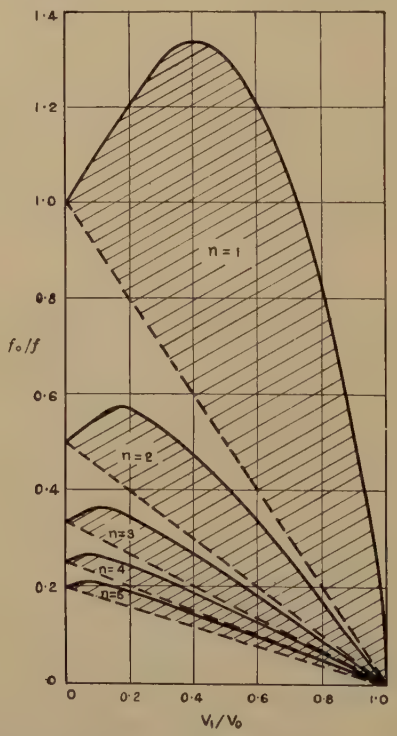

Fig. 6—The synchronization of a simple relaxation oscillator.[12]
f = control frequency
f_0 = natural relaxation frequency
V_1 = control voltage
V_0 = an oscillator constant
n = order of frequency division.
Ranges of f_0/f over which synchronization occurs are shaded.

with the experimental conditions. In these curves it is to be noted that the range of synchronization is symmetrical about the ratio f_0/f (equal to the reciprocal of the order of frequency division) only for very small control voltages. As the control voltage is increased the upper limit of f_0/f is rapidly reduced and then falls below the reciprocal of the order of frequency division; i.e., synchronization then occurs only if the natural relaxation frequency is less than the required synchronized frequency.

IV. Alternative Stabilizing Circuits

The basic functions required in the method of stabilization illustrated in Fig. 1 are substantially independent in the operation of the circuit of Fig. 2, and this facilitates analysis and design. There are, however, advantages to be gained in some practical cases by reducing the number of circuit components required, even if the circuit operation is thereby made more complex. A wide range of circuits achieving more or less economy in this respect could obviously be worked out; that shown in Fig. 7 is given as one example.

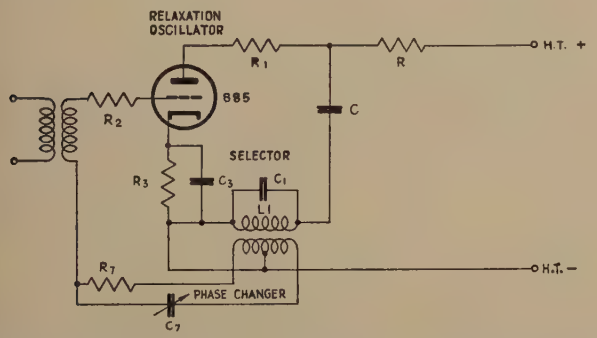

Fig. 7—A simplified circuit for stabilization of a relaxation oscillator. The functions indicated by Fig. 1 are no longer independent.

In the arrangement of Fig. 7, an isolating valve amplifier is not used and the tuned selector circuit L_1C_1 has been connected in series with the relaxation condenser C; some degree of separation of the functions of relaxation and selection then requires a choice of the constants L_1C_1 and C in such a way as to keep the oscillatory voltage developed across the selector appreciably less than the relaxation voltage at the anode of the gas-triode valve. Other elements of the relaxation oscillator and the phase-changing network in the feedback circuit can be substantially the same as in the circuit of Fig. 2. The operation of the circuit is indicated by the generated wave forms shown in Fig. 8. The voltage e_a between the anode of the gas triode and earth departs from the usual saw-tooth relaxation form, owing to the voltage e_s developed across

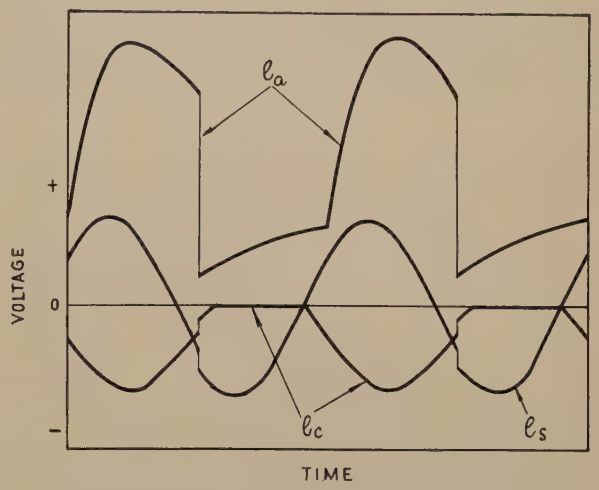

Fig. 8—Wave forms of voltages generated in an oscillator using the circuit of Fig. 7.

the selector L_1C_1. The voltage e_c between the grid and the cathode of the valve provides the feedback necessary for stabilization. The small discontinuities in the substantially sinoidal wave form e_s, due to the critical potential of the relaxation oscillator and the limited selectivity of the selector, again indicate suitability for high-order frequency division. The operation is thus substantially the same as that of the circuit of Fig. 2; but it is more complex to analyze in detail and approaches more closely the operation of conventional power-inverter circuits.

V. Acknowledgments

The investigation described in this paper was carried out as part of the program of development of Amalgamated Wireless (Australasia), Ltd., and I am indebted to the Company for permission to publish the results. I am also indebted to Mr. N. F. Roberts, for obtaining the experimental data summarized in Figs. 4 and 5.

Contour-Mode Vibrations in *Y*-Cut Quartz-Crystal Plates*

GEOFFREY BUILDER†, ASSOCIATE, I.R.E., AND J. E. BENSON†, ASSOCIATE, I.R.E.

Summary—*Theoretical considerations and experimental results both indicate that the principal contour-mode responses in Y-cut quartz plates, excited by simple electrodes on the major faces, are due to coupling between the fundamental shear mode and the second flexural mode and its harmonics. Extensional and first flexural modes can be excited only by the use of special electrode arrangements.*

I. Introduction

IT IS well known that a quartz-crystal plate may vibrate at frequencies determined by the edge or contour dimensions as well as at higher frequencies determined primarily by the thickness of the plate. The lower-frequency contour modes are important because they may be utilized in low-frequency resonators, and also because their existence interferes to a marked extent with the thickness-mode oscillation, which is used extensively in higher-frequency resonators. The study of these contour modes has therefore important practical advantages apart from its considerable academic interest. Owing to the multiplicity of modes of vibration possible in any quartz plate, a complete analysis presents formidable difficulties. The contour modes of an *X*-cut plate have been analyzed in some detail by Mason;[1] but there is little definite information about these modes in a *Y*-cut plate, and a number of earlier statements appear to be obscure, if not incorrect.

It has been implied by Lack[2] and later stated by Hund,[3] Modrak,[4] and others that the principal contour mode in a *Y*-cut plate, when excited by an electric field normal to its major faces, is an extensional vibration in the direction of the electric or *X* axis. Hight and Willard[5] have, on the other hand, indicated that the contour modes in *V*-cut plates[6,7] are pure shear modes in the plane of the crystal plate, and this suggests that the *Y*-cut plate, regarded as a special case for which the *V* angle is zero, would exhibit the same type of vibration.

The nature of a vibrational mode can be investigated by analyzing the dependence of its frequency on the ratio of the significant dimensions of the crystal element; this method was used by Mason[1] in his analysis of the contour modes of an *X*-cut plate. An analysis of the piezoelectric and elastic equations for quartz also indicates the nature of the modes of vibration to be expected with different methods of excitation, and in some cases enables the frequencies of these modes to be calculated. We have used these methods to investigate the nature of the contour modes in *Y*-cut plates, and present the results obtained in this paper. There does not appear to be any evidence for the existence of a longitudinal mode when the crystal is excited in the usual way by means of electrodes on the major faces; on the other hand, both theoretical considerations and experimental observations indicate that these contour modes are due to shear and flexural vibrations and to coupling between them.

The piezoelectric effect in quartz may be stated in terms of strains produced by electrical stresses, providing there are no external forces; the piezoelectric equations, referred to a set of principal axes, take the form

$$\left.\begin{aligned} x_x &= d_{11}E_x & y_z &= d_{14}E_x \\ y_y &= -d_{11}E_x & z_x &= -d_{14}E_y \\ z_z &= 0 & x_y &= -2d_{11}E_y \end{aligned}\right\} \quad (1)$$

where E_x, E_y, E_z are the electrical stresses, x_x, x_y, z_z extensional strains, y_z, z_x, x_y shear strains, and d_{11} and d_{14} are the piezoelectric constants. The strains indicated by (1) may lead to a complex system of coupled responses determined by the boundary conditions and the elastic properties of the quartz. The elastic equations[8,9] are:

$$\left.\begin{aligned} -X_x &= C_{11}x_x + C_{12}y_y + C_{13}z_z + C_{14}y_z \\ -Y_y &= C_{12}x_x + C_{22}y_y + C_{23}z_z + C_{24}y_z \\ -Z_z &= C_{13}x_x + C_{23}y_y + C_{33}z_z \\ -Y_z &= C_{14}x_x + C_{24}y_y + C_{44}y_z \\ -Z_x &= C_{55}z_x + C_{56}x_y \\ -X_y &= C_{56}z_x + C_{66}x_y \end{aligned}\right\} \quad (2)$$

in which X_x, Y_y, etc., are the mechanical stresses referred to the principal axes and the *C*'s are the elastic moduli.

II. General

Equations (1) and (2) define the response of a quartz element to piezoelectric excitation. A complete analysis

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† Amalgamated Wireless (Australasia), Ltd., Ashfield, N.S.W., Australia.

[1] W. P. Mason, *Bell Sys. Tech. Jour.*, vol. 13, p. 405; 1934.

[2] F. R. Lack, "Observations on modes of vibration and temperature coefficients of quartz crystal plates," PROC. I.R.E., vol. 17, pp. 1123–1141; July, 1929.

[3] A. Hund, "Phenomena in high frequency systems," chapter 8, McGraw-Hill Book Co., New York, N. Y., 1936.

[4] P. Modrak, *Wireless Eng.*, vol. 16, p. 6; 1939.

[5] S. C. Hight and G. W. Willard, "A simplified circuit for frequency substandards employing a new type of low-frequency zero-temperature coefficient quartz crystal," PROC. I.R.E., vol. 25, pp. 549–563; May, 1937.

[6] C. F. Baldwin and S. A. Bokovoy, Australian Patent No. 21,957/35, 1935.

[7] G. Builder and J. E. Benson, *A.W.A. Tech. Rev.*, vol. 3, p. 157; 1938.

[8] W. Voigt, "Lehrbuch der Kristallphysik," first edition, Teubner, Leipzig, Germany, 1910. (Later edition 1928.)

[9] A. E. H. Love, "The Mathematical Theory of Elasticity," fourth edition, Cambridge University Press, Cambridge, England, 1934.

is not practicable owing to the complexity of the response, but a number of qualitative conclusions may be derived from the equations, and in special cases the frequencies of the vibrational modes may be calculated with some accuracy, using empirical values of the piezoelectric constants and the elastic moduli. Work along these lines together with experimental measurements of response frequencies has already yielded much information, and that relevant to the present discussion will be briefly summarized.

Extension Modes

Equation (1) indicates that extensions are caused directly by the piezoelectric effect only when there is a component of electric force along the X axis; electrical stresses along the Y or Z axes do not cause any extensional strains parallel to a principal axis. It is

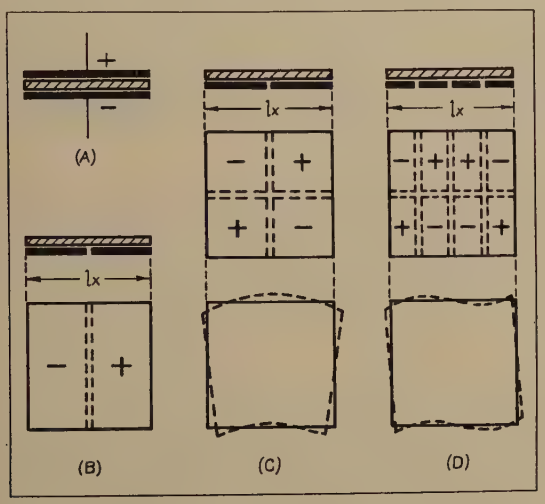

Fig. 1—Arrangement of electrodes applied to Y-cut plates for exciting
(A) x_y and x_z shear modes
(B) x_x extension mode
(C) Fundamental flexure in the X–Z plane
(D) Second flexure in the X–Z plane.
In (C) and (D) the general form of the flexures is also indicated.

therefore to be inferred that in a Y-cut crystal plate, i.e., one in which the major faces are normal to the Y axis, electrical stresses between electrodes on the major faces (Fig. 1 (A)) would fail to excite directly an extensional vibration. Moreover, the shear strains z_x and x_y produced by an electric force along the Y axis according to (1) cannot excite extensional modes by mechanical coupling, as is indicated by the last two of equations (2). There is, therefore, no evidence from the equations of any extensional mode excited in a Y-cut plate by electrical stresses applied between simple electrodes on the major faces of the plate.

The extensional strains x_x and y_y may be produced directly by an electrical stress E_x along the X axis. These extensions, the shearing strain y_z also produced by E_x, and the extensional strain z_z along the optical axis, are mechanically coupled in accordance with the first four of equations (2). The responses in an X-cut plate having electrodes on the major faces are thus indicated and have been analyzed in some detail by Mason.[1] Similar responses in a Y-cut plate may be produced by electrodes arranged to apply an electrical stress E_x; the electrodes may be applied to the edges of the plate normal to the X axis, or as in Fig. 1 (B) to produce surface stresses in the manner of Giebe and Scheibe.[10]

Apart from such special electrode arrangements extensional vibrations are, according to (1) and (2), not to be expected in a Y-cut plate. It is, however, clear that, if the Y-cut plate is not oriented accurately, a component of electrical stress along the X axis may be produced by the normal electrode system of Fig. 1 (A); we have in practice observed weak extensional responses on this account, but they have not been detectable in plates cut accurately normal to the Y axis

Shear Modes

An electrical stress E_y, applied to a Y-cut plate by the usual electrode arrangement of Fig. 1 (A), causes the shear strains z_x and x_y in accordance with (1), and these strains are mechanically coupled one with the other in accordance with the last two of equations (2). The vibration corresponding to the strain x_y is the principal "thickness-mode" of a Y-cut plate and has been widely utilized for high-frequency oscillators and resonators. The strain z_x in the plane of the major faces of the Y-cut plate is a pure shear[9] and the corresponding vibration has a frequency determined by the major dimensions of the plate; it is thus obviously the chief contour mode of oscillation.

Following Mason,[1] the frequency of the fundamental of a shear vibration may be expressed as

$$f_s = (K_0/a)\sqrt{1 + (a/b)^2}, \tag{3}$$

where

$K_0 = \frac{1}{2}\sqrt{C_{ij}/\rho}$,

C_{ij} = the appropriate elastic modulus,

ρ = the density of quartz,

a and b = the contour dimensions of the plate.

K_0 is the frequency for a thin bar of unit length along the a dimension, and may be called the frequency constant of the mode. By substituting the appropriate values of the constants, the frequency in kilocycles of the contour shear vibration corresponding to the strain z_x becomes

$$\left.\begin{aligned} f_x &= (231/l_x)\sqrt{1 + l_x^2} \qquad l_z = 1 \\ f_z &= (231/l_z)\sqrt{1 + l_z^2} \qquad l_x = 1 \end{aligned}\right\} \tag{4}$$

where l_x and l_z are the dimensions, in centimeters, of a thin plate, along edges parallel to the X and Z axes, respectively. It is to be observed that the frequency is the same for a given shape of crystal of fixed major

[10] E. Giebe and A. Scheibe, *Zeit. für Phy.*, vol. 46, p. 607; 1928.

dimension whether this dimension is along the X axis or along the Z axis.

Flexural Modes

Mason has pointed out that the distortion of an element of a plate vibrating in a pure shear mode is similar to that when vibrating in a second flexural mode, as shown in Fig. 1 (D). Mechanical coupling between the two modes, therefore, is to be expected, and was found by Mason to occur in X-cut plates;

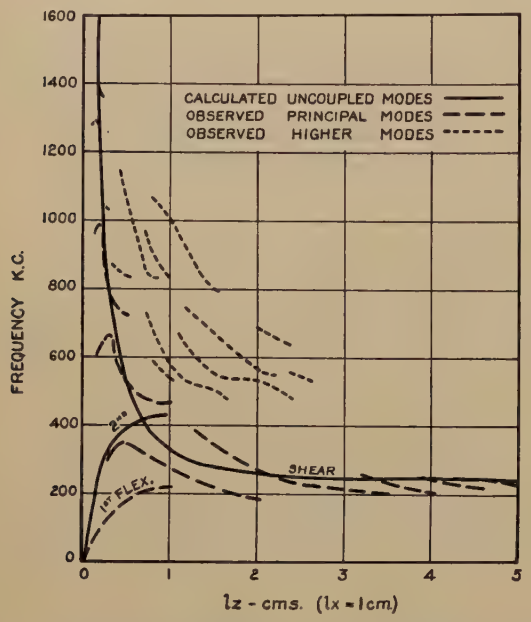

Fig. 2—Contour-mode responses in thin Y-cut plates having a dimension along the X axis of 1 centimeter.

flexural modes must therefore be considered in the case of a Y-cut plate in which the shear mode is normally excited by the piezoelectric effect.

Flexural modes may be excited piezoelectrically by electrode arrangements suitable to the production of simultaneous contraction and extension of a pair of opposite edges. In a Y-cut plate such extensions must be produced, as discussed above, by electrical surface stresses, or by electrodes applied to the edges of the crystal plate, to obtain electrical stresses having a component along the X axis. A fundamental flexure may be excited directly by the electrode arrangement shown in Fig. 1 (C), in which the nature of the distortion of the plate is also indicated. The electrode arrangement of Fig. 1 (D) is appropriate to the excitation of the second flexural mode; it is not to be expected that this mode can be isolated and its frequency measured if, as is anticipated, it is mechanically coupled to the z_x shear.

The frequency of a bar vibrating in flexure has been calculated by Mason,[11] including corrections necessary for rotary and lateral inertia in bars having widths comparable with their lengths. Earlier calculations[12] were based on the well-known formula[13,14]

$$f = Kb/l^2 \tag{7}$$

for a very thin bar of width b and length l, K being a constant depending on the elastic constants of the material. Harrison had, by means of empirical corrections, used this formula for ratios of width to length up to 0.25, but extrapolation of his formula to wider bars results in considerable errors. Mason's formulas are applicable to both first and second flexural modes, and he has used them to calculate the frequency of the first flexure for bars in which $l_z/l_y < 0.5$ and of the second flexure for bars in which $l_z/l_y < 0.3$. These calculations agree with observed values within about 1 per cent.

Tykociner and Woodruff[15] have investigated both X- and Y-cut bars and have shown that the flexural frequencies are identical when the width of both is along the Z axis. This is to be expected, as Young's modulus has the same value along the Y and X axes and these coincide respectively with the major axes of the two types of bars.

The calculated frequencies of the first and second flexural modes of a Y-cut bar have been plotted in Fig. 2 for values of $l_z/l_x \leqq 1$; these are the same as the values calculated by Mason for X-cut bars. The curves for the flexural mode were checked by direct measurements.

III. Experimental Results

The experimental investigation of the contour modes of Y-cut plates took the form of an extensive series of measurements of the contour-mode frequencies of thin plates having rectangular major faces with edges parallel to the X and Z axes. The plates were all thin enough to avoid any appreciable effect of the "thickness mode," i.e., the y_z shear, on the contour-mode oscillations, and to make the frequency of the thickness mode much higher than the range of contour-mode frequencies to be measured. The contour-mode frequencies were measured for a series of ratios of the edge dimensions of the rectangular major faces, i.e., ratios of the dimensions l_x and l_z along the electric and optic axes.

It is well established that, for a crystal element of fixed shape, all the response frequencies are inversely proportional to any one dimension of the element. For convenience in the experimental work, and to avoid working on excessively small elements, measurements were made on elements of convenient size, and the measured frequencies were converted to the corre-

[11] W. P. Mason, *Jour. Acous. Soc. Amer.*, vol. 6, p. 246; 1935.

[12] J. R. Harrison, "Piezo-electric resonance and oscillatory phenomena with flexural vibrations in quartz plates," Proc. I.R.E., vol. 15, pp. 1040–1054; December, 1927.

[13] Lord Rayleigh, "Theory of Sound," second edition, vol. 1, chapter 8. Macmillan, London, England, 1894.

[14] E. H. Barton, "Textbook on Sound," Macmillan, London, England, 1908.

[15] J. T. Tykociner and Marion W. Woodruff, *Univ. of Illinois Eng. Exp. Sta., Bull.*, No. 291, 1937.

sponding frequencies for plates in which the X dimension was fixed at 1 centimeter. The results of the measurements after this conversion are summarized in the broken-line curves of Fig. 2, which show the frequencies of various oscillation modes for values of l_z from about 0.1 to 5 centimeters. The full-line curves show the calculated frequencies of the fundamental x_z shear and the second flexural mode. The experimental curve for the first flexure was obtained with the appropriate electrode assembly of Fig. 1 (C), and agrees closely with the values calculated following Mason.[11] The experimental curves were all carefully checked by measurements on a multiplicity of crystals of different rectangular shapes and of various relative thicknesses, the measurements agreeing within the limits of experimental error and showing that the results were independent of the relative thickness of the plates used.

The calculated shear and second-flexure frequencies shown by the full-line curves of Fig. 2 were not observed experimentally; but the curves for the principal modes observed suggest strongly that they are due to coupling between the shear and second flexural modes. This interpretation of the results is clarified by replotting the curves after the manner of Lorenzen[16] as shown in Fig. 3; as before, ordinates represent frequencies and abscissas represent ratios of dimensions along the X and Z axes. The origin of co-ordinates represents a dimensional ratio of unity; to the left of the origin are plotted values of $l_z<1$ for $l_x=1$, and to the right values of $l_x<1$ for $l_z=1$. Logarithmic abscissas have been adopted in order to open the scale, where most desired, at ratios tending to zero. The symmetry implied in (3) and (4) for the shear mode frequency is apparent from this graph. The curves show, as before, the calculated shear and second flexure frequencies as well as the calculated first flexure frequency for $l_x/l_z<1$. The principal frequencies observed are shown in the broken-line curves, and higher observed frequencies in dotted curves. The general symmetry about the dimensional ratio of unity is apparent when the results are presented in this form.

It is clear that the principal observed frequencies, for $l_z<l_x$, arise from the coupling between the first shear mode and harmonics of the second flexure. The higher observed frequencies appear to be due to coupling between the second harmonic of the shear and harmonics of the second flexure, but these have not been analyzed in detail. The first flexure for $l_x<l_z$ could not be observed experimentally for the reasons set out in section II above. The slight asymmetry of the first and second flexures about the unity dimensional ratio, and the corresponding slight asymmetry of the principal observed frequencies, is ascribed to the somewhat higher value of the elastic modulus along the Z-axis (10.1×10^{11} dynes per square centimeter) than in the XY plane (7.7×10^{11} dynes per square centimeter). The discontinuities in the curves for the observed frequencies, and the general form of the curves, are in close accord with those to be expected as a result of coupling between modes.[2]

[16] R. Lorenzen, *Communications*, vol. 19, 1, p. 7; 1939.

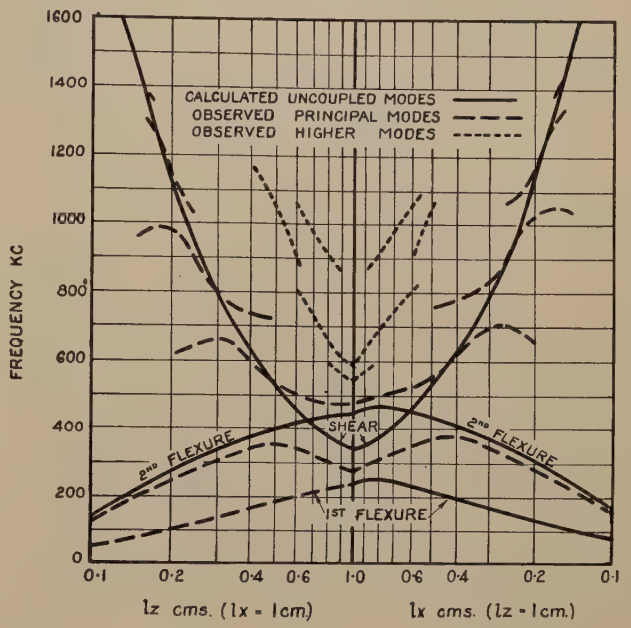

Fig. 3—Symmetrical plot of contour-mode responses in thin Y-cut plates. The curves show the change of frequency of the various modes of a plate, initially square, as either dimension is progressively reduced, the other dimension remaining at 1 centimeter.

Thus it appears permissible to conclude that the principal contour-mode responses of Y-cut plates are due to coupling between the shear and the second flexure and its harmonics, and that the excitation of extensional and first flexural modes occurs only with special electrode arrangements.

IV. Acknowledgment

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Acoustic Models of Radio Antennas*

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Summary—The large number of independent variables in antenna arrays make it advisable to develop means for rapidly surveying the field patterns which may be obtained. The advantages in the use of acoustic models for this purpose are shown.

Two analogous acoustic antennas are developed, one for the measurement of fields at a distance and the other for fields close to the antenna. The procedure which can be followed in setting up the models and making measurements is outlined.

Advantages are demonstrated for the acoustic model in the study of nonsinusoidal current distributions, and their effect on the field pattern.

The measurement of mutual impedance between antennas may be conveniently carried out with one of the models. The arrangement for phasing and magnitude control in a multiunit array are shown. The field of an acoustic model is shown in comparison with the field of an actual array which it simulates.

Introduction

THE analysis of the field pattern of any given combination of radiating elements in an antenna array may be accomplished by straightforward, but lengthy, calculations. The determination of the optimum combination of these independent variables to accomplish a desired result becomes correspondingly cumbersome because of the number of the parameters. The independent variables whose values must be selected by the designer are 1, the number of elements; 2, the height of each element; 3, the geometrical arrangement of the element; 4, the magnitude of current in each element; 5, the phase of the current in each element; and 6, the distribution of the current in each element.

Because of the work involved in making the computations, the possible combinations which are available in antenna arrays have not been completely explored.

In power networks there are also available straightforward methods for the computations of the conditions which are produced by particular combinations. However, the length of the calculations required for any single possible configuration has given rise to the development of network analyzers. The analyzers can simulate actual conditions with a few adjustments on a model and obtain answers very rapidly with the required degree of approximation.

Models of antennas have been found useful in many studies. These models have been constructed by operating short antennas at frequencies high enough so that the dimensions in wavelengths are the same as for the structures which they are simulating. At these high frequencies the difficulties of making measurements without disturbing the field are greatly increased. Furthermore it is not easy to simulate the ground plane at these frequencies because the reflection characteristics of the earth are a function of frequency.

The dimensions of a radiating system, in wavelengths, may be reduced by increasing the frequency or by reducing the velocity of propagation. Since all wave motion has certain common characteristics, wave phenomena may be studied by disturbances propagated in any media. If acoustic waves in air are used in place of electromagnetic waves, the velocity of propagation is reduced almost a millionfold. The phenomena of interference, which are the determining factors in directional-antenna systems, can be studied with equal facility by means of these acoustic waves. The dimensions of the system can then be reduced even further than is feasible with present high-frequency techniques. At 600 cycles the wavelength in air at normal pressure and a temperature of 20 degrees centigrade is 22.6 inches. At this frequency field measurements may readily be made on acoustic waves by means of a microphone, a calibrated audio-frequency amplifier, and a rectifier voltmeter.

In drawing the analogy between electromagnetic waves and acoustic waves, account must be taken of the fact that the former are polarized while the latter are compressional waves. This can be done by relatively simple means which will be pointed out later.

The acoustic antenna analyzer makes use of sound radiators which set up wave patterns similar to those of radio antennas. Corresponding independent variables can be controlled by simple adjustments. In order to illustrate the method and the results the effects obtained with analogies to known antenna systems will be shown.

Fundamental Analogy

The equation for the pressure set up by a point source of sound is

$$p = \frac{P_1}{r} e^{-jkr} \tag{1}$$

where p = the increment of pressure at any point in the medium,
P_1 = the pressure at unit distance,
r = the distance of the point from the source,
and $k = 2\pi$ per wavelength of the wave.

The equations for the components of the electric field at any point produced by a doublet of length dl carrying a current I are

$$E_\theta = Idl \sin\Theta \left[\frac{-j}{\omega r^3} + \frac{1}{r^2 c} - \frac{j\omega}{rc^2} \right] e^{-jkr} \tag{2}$$

$$E_r = 2Idl \cos\Theta \left[\frac{-j}{\omega r^3} + \frac{1}{r^2 c} \right] e^{-jkr} \tag{3}$$

where r = the distance of the point from the center of the doublet,
θ = the angle between r and the axis of the doublet, and
c = the velocity of electromagnetic waves.

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These expressions show that the electric field due to such a doublet consists of both radial and tangential components. However, at large distances the term which contains r to the first power in the denominator is the only one of importance and is called the radiation term. Therefore at large distances the electric field is given by the relation

$$E = E_\theta = -jIdl \sin \Theta \frac{\omega}{rc^2} e^{-jkr} \qquad (4)$$

or, if E_1 is the value of E_θ at a unit distance for $\theta = 90$ degrees,

$$E = E_\theta = \frac{E_1}{r} \sin \Theta e^{-jkr}. \qquad (5)$$

It is apparent that (5) is similar to (1) except for the term sin θ in the former. This term is due to the polarization of electric waves, an effect which is not present in the pressure of a sound wave. All measurements made on the acoustic fields must be corrected by the sin θ term to obtain analogous radiation fields for an electric radiator.

The Simple Vertical Antenna

In a radio antenna the radiation can be considered to be emanating from elemental doublets distributed along the antenna. In order to obtain an analogous line distribution of sound radiation, a series of holes may be drilled along a pipe. A sound pressure should then be developed behind each hole proportional to the current in corresponding portions of the antenna. In the models the pipes used were excited from Western Electric 555W receivers driven by an oscillator and amplifier.

The current distribution in an unterminated radio antenna is a standing wave. If the cross section of the antenna is uniform this standing wave is approximately a sine wave in space along its length, starting with a zero value at the upper end. In order to obtain a similar distribution of *pressure* within a pipe the pipe should be open at the end. This would cause a considerable radiation of sound from the open end which would invalidate the results. The same effect can be obtained by closing the end of the pipe and starting the distribution of the holes one-quarter wavelength from the closed end where a node of pressure would exist. A half-wave antenna would then have a series of drilled holes separated by short intervals and covering a length of 11.3 inches when a frequency of 600 cycles is to be used.

The radiation of energy through the holes will produce an attenuation of the wave which prevents the nodal points of pressure from being zero, just as the current on an antenna is not zero at the nodes. The magnitude of the pressure at the nodes is affected by the relative diameters of the holes and the pipe.

Before proceeding with the antenna proper a study was made of the pressure along the pipe. This pressure was obtained by a stethoscope arrangement consisting of a small rubber tube leading to an enclosed chamber containing a crystal headphone which in turn was connected through the calibrated amplifier to the voltmeter. The holes along the pipe in the first test were 1/64 of an inch in diameter spaced 1 inch apart. Fig. 1

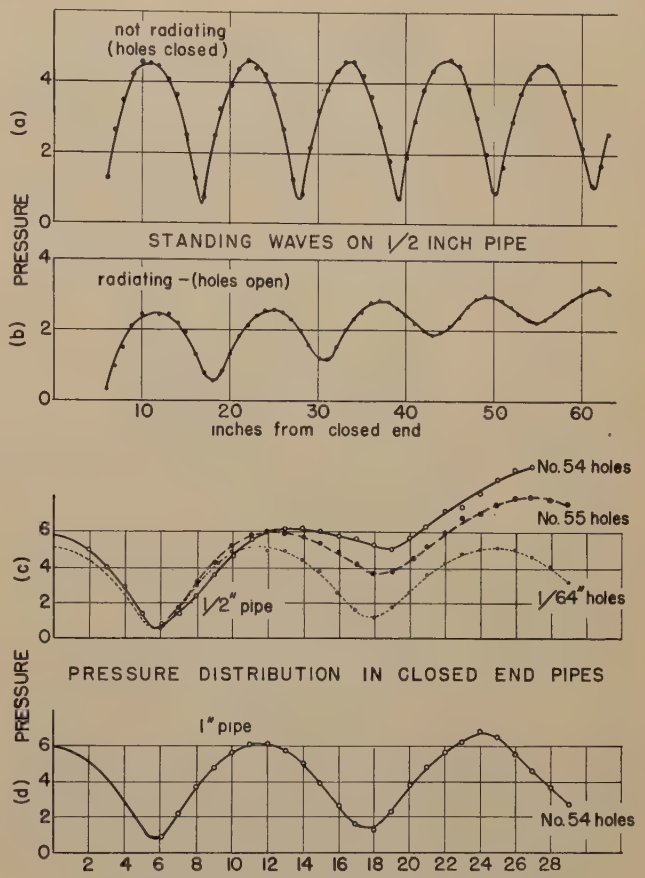


Fig. 1—Pressure distribution along long pipes with standing acoustic waves and radiating holes.

shows results on a long pipe. Curve *a* shows the case where all holes except the one at the measuring point were plugged up, so that the distribution is that of a standing wave with only the losses due to absorption by the pipe walls. Curve *b* shows the case where all holes are allowed to radiate. Two modifications due to this radiation are observable. One is that the attenuation has materially increased the magnitude of the pressure at the nodes and the second is that the phase velocity has increased above that which holds in air. The holes produce an effect similar to shunt inductance and resistance on a line. Such a shunt inductance would increase the phase velocity on a line and so the holes increase the phase velocity in the pipe.

Curve *c* shows the pressure distribution on a $\frac{1}{2}$-inch pipe for different sized holes, and it is seen that as the diameter of the holes is increased, the distribution of the pressure departs more and more from the sinusoidal function. Curve *d* shows the effect of increasing the diameter of the pipe while keeping the size of the holes constant, and it is seen that the distribution of

the pressure is more nearly sinusoidal and the phase velocity is more nearly that in air.

The departure from sinusoidal distribution due to dissipation by radiation also exists on an actual antenna. Where information on the current distribution

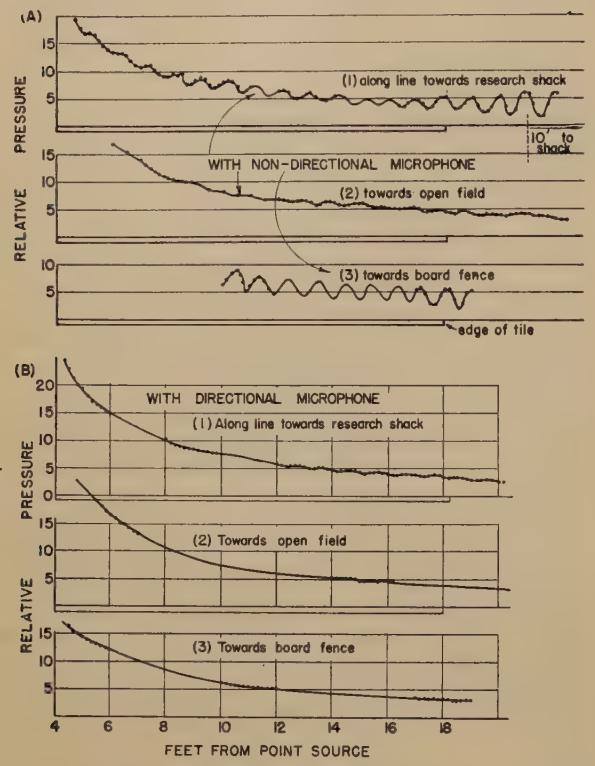


Fig. 2—Disturbing effect of reflecting bodies in an acoustic field and elimination by directional microphone.

of an actual antenna is available the sizes of pipe and holes can be adjusted to produce an analogous result. This will be discussed in more detail later. The patterns which follow were in general obtained by the use of 1-inch pipe and a No. 54 drill (0.0550 inch).

In determining the field about an acoustic antenna, the microphone was arranged so it could be moved

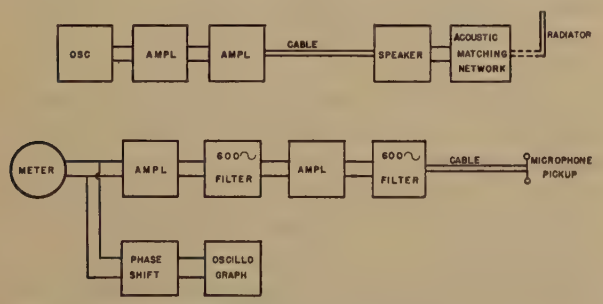


Fig. 3—Block diagram of electrical-measuring equipment for acoustic-antenna model.

about the source at a constant radius with both the polar and azimuth angles variable. An alternative method is to rotate the antenna and keep the pickup fixed. Since measurements are desired at only one frequency, a selective circuit was introduced between the pickup and the amplifier. This selectivity served to reduce the effect of extraneous noises to a negligible value and also cut out any harmonics which might be generated by the sound source.

The model antenna was arranged to project above a reflecting surface by an amount corresponding to the height of the antenna being simulated. It was thought at first that the reflecting surface could be a platform of a few wavelengths in each dimension and the excitation obtained from below. It was found that the finite character of the platform produced a marked effect on the field distribution due to diffraction at the edges. For that reason the reflecting surface was transferred to the ground where a tile surface was laid out with a diameter of 35 feet. The plane surface of the tile was continued in the ground beyond. Hence the diffraction

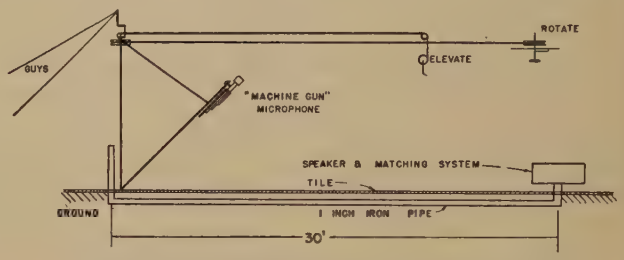


Fig. 4—Physical arrangement of acoustic antenna and pickup microphone.

at the edges was eliminated and reflection at the edge was negligible.

In the first measurements the microphone used was a simple crystal headphone. However the results obtained for the field of simple radiating sources did not show circular symmetry. It was found that this was due to reflections from objects in the vicinity which produced standing-wave patterns. This is demonstrated by measuring the sound-field distribution along radial lines. Fig. 2 shows the measurements along three such radials with the simple microphone. If no disturbing conditions exist, the field strength should be inversely proportional to the distance from the source. The effect of reflections from bodies in the sound field is very marked. A unidirectional microphone was developed which operated upon directional-antenna principles and radial measurements again made. This eliminated the sensitivity of the pickup to the waves reflected from disturbing bodies and the smooth curves shown for this case in Fig. 2 show this to be the solu-

Fig. 5—Photograph of acoustic antenna and microphone boom.

tion of this problem. Since the measurements are made at only one frequency, the microphone does not require a flat frequency characteristic.

The block diagram of the complete measuring apparatus is shown in Fig. 3 and the physical arrangement is outlined in Fig. 4. In order to remove the disturbing effect of an observer, the boom supporting the microphone was arranged so that it could be rotated and elevated from a control point some 50 feet from the acoustic antenna. The measuring equipment was also located at some distance and connected by shielded cables to the microphone and loud-speaker unit. Fig. 5 is a photograph of an antenna system with the boom and microphone. A close-up of a single vertical antenna is shown in Fig. 6.

The results obtained using the antenna of Fig. 6 are shown in Fig. 7. Measurements were made and compared with theoretical computations for such an antenna for two different heights. The agreement is so good that the use of these models in more complicated antenna arrays is justified. The two sets of curves are compared on a basis of equal fields along the horizon and not on equal radiated powers.

Effect of Nonsinusoidal Current Distributions

When calculations are made on an antenna system to determine its directional characteristics, a sinusoidal current distribution is generally assumed. To attempt to account for the nonsinusoidal character of the actual distribution complicates the problem greatly and makes the amount of the computational work excessive. For antennas less than a half wavelength in height, the error introduced by the assumption is negligible. For antennas of greater height the assumption may be responsible for a few degrees shift in the maxima and minima of the horizontal pattern. However, at the higher angles, for these large antennas, the nonsinusoidal distribution may markedly affect the pattern and some account must be taken of it.

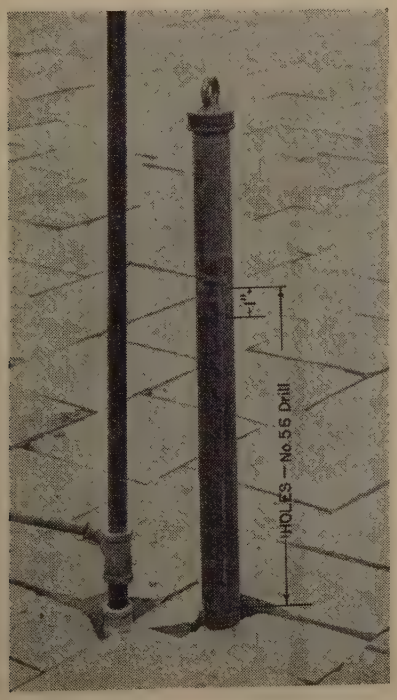


Fig. 6—Close-up of antenna model and boom support.

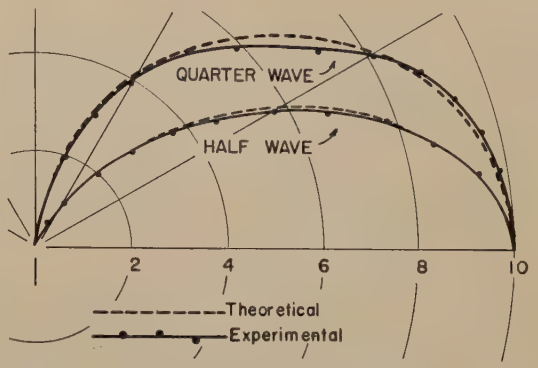


Fig. 7—Field patterns of single vertical antenna.

In the electric antenna, where the radiator is of uniform cross section, the departure from the sinusoidal distribution is caused primarily by the losses in the system, of which the largest is due to radiation. The pressure distribution on an acoustic antenna is also nonsinusoidal and for a similar reason, i.e., the losses due to the radiation from the holes in the pipe. The question to be answered is: "Is the nonsinusoidal pressure distribution on an acoustic antenna the same as the nonsinusoidal current distribution on the electric antenna?" Since almost any distribution of magnitude may be obtained on the model antenna by a suitable choice of hole sizes, this question reduces to the following: "When the magnitude of the pressure on the model corresponds at all points to the magnitude of the current on the actual antenna, will the phase of the pressure vary in a manner similar to the phase variation of current?"

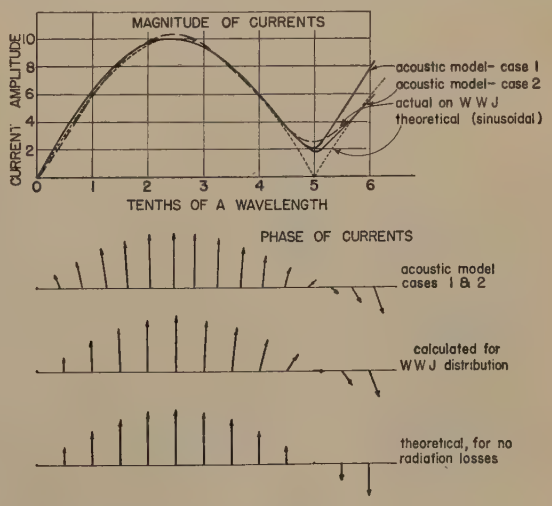


Fig. 8—Distribution of current along an electrical antenna compared with distribution along an acoustic antenna.

The phase and magnitude of the currents on an antenna were computed by long-lines theory, and it

was found that the same equations applied to both the acoustic and electric radiators. Morrison and Smith[1] have measured the current distribution along a uniform cross-section tower at station WWJ. A model of this antenna was set up and the phase and magni-

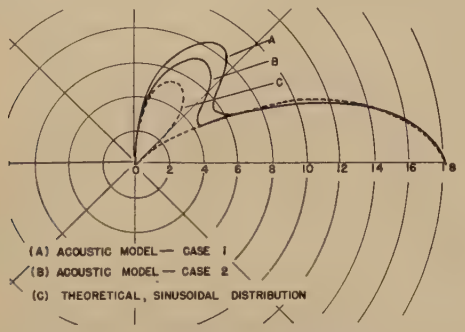


Fig. 9—Comparison of patterns of vertical antennas with sinusoidal and nonsinusoidal distributions.

tude of the pressure compared with the values computed by the long-lines theory. The hole size was so selected as to correspond in one case to the WWJ distribution. The results for the calculations and measurements are shown in Fig. 8 and are in excellent agreement. The result of this nonsinusoidal distribution on the vertical pattern is shown in Fig. 9. It will be observed that there is a marked divergence of the pattern from the sinusoidal case at the higher angles. The change in magnitude is accompanied by a large shift in the phase of the field at the higher angles. If such an antenna is used as an element of a directional array the deviation of the field pattern from that computed for sinusoidal distribution would be very marked at the high angles. The acoustic model would give the actual polar field pattern.

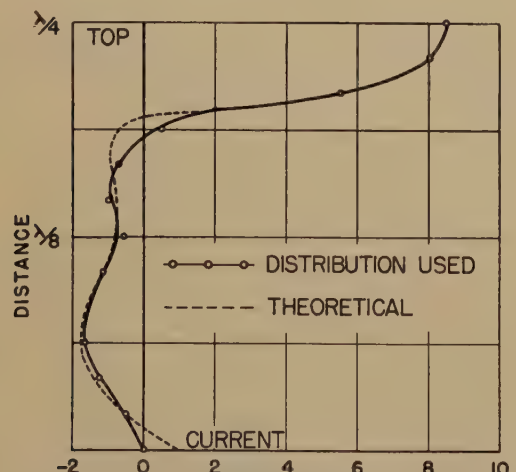


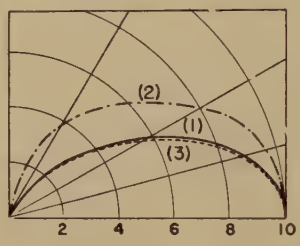
Fig. 10—Optimum current distribution for a quarter-wave vertical antenna.

Special Current Distributions

The acoustic-model method is readily applicable to special current distributions. For example the effect of top loading can be simulated by plugging up some of the holes below the pressure node so that the pressure of the top hole corresponds to the current into a "hat."

Geoffrey Miller, formerly at The Ohio State University, studied the optimum distribution of current on a vertical antenna by the use of the calculus of variations. He found that a current distribution on a quarter-wave vertical antenna similar to that shown in Fig. 10 would give a polar field which is practically the same as a half-wave antenna with a sinusoidal current distribution.

In advance of the study of methods for securing such a distribution on a radio antenna, the effect may readily be studied by the acoustic antenna. With the acoustic antenna a special current distribution can be simulated by drilling holes of different sizes along the pipe. The pressure distribution is then checked by the stethoscope method. The pressure distribution which was secured by this means is also shown in Fig. 10. The corresponding polar diagram is shown in Fig.



(1) 1/4 WAVE, OPTIMUM CURRENT DISTRIBUTION
(2) 1/4 WAVE, SINUSOIDAL DISTRIBUTION
(3) 1/2 WAVE, SINUSOIDAL DISTRIBUTION

Fig. 11—Comparison of vertical patterns for optimum current distribution and sinusoidal current distribution.

11. For comparison the diagrams of standard half- and quarter-wave antennas are also shown and the conclusions of Miller are verified.

Alternate Analogy and Model for a Vertical Antenna

The expression for the component of the electric field parallel to an antenna with a sinusoidal space distribution of current on the antenna is

$$E_z = j\frac{I_0}{c}\left[\frac{2\cos kl\, e^{-jkr_0}}{r_0} - \frac{e^{-jkr_1}}{r_1} - \frac{e^{-jkr_2}}{r_2}\right] \quad (6)$$

where

E_z = the component of the field in statvolts per centimeter parallel to the antenna at the point p,
I_0 = the current in abamperes at the antinode or loop of the antenna,
r_0 = the distance from the point p to the base of the antenna,
r_1 = the distance from the point p to the top of the antenna,
r_2 = the distance from the point p to the image of the top of the antenna,
$k = 2\pi/\lambda = \omega/c$, and
l = the length of antenna.

[1] J. F. Morrison and P. H. Smith, "The shunt-excited antenna," Proc. I.R.E. vol. 25, pp. 673–696; June, 1937.

The geometry is illustrated in Fig. 12.

Equation (6) indicates that the field in the z direction behaves as though the disturbance originated at the two ends of the antenna. The disturbance originating at the top is propagated over two paths. One of these paths is r_1 and the other is the reflection from the ground plane corresponding to the path r_2. Therefore the field of the antenna could be simulated by two point sources, one at the surface of the ground plane and the other at the top of the antenna. Furthermore (6) corresponds to a field due to the point sources of two spherical waves, and no correction need be made for polarization if acoustic waves are used to find the z component of the electric field. Therefore an

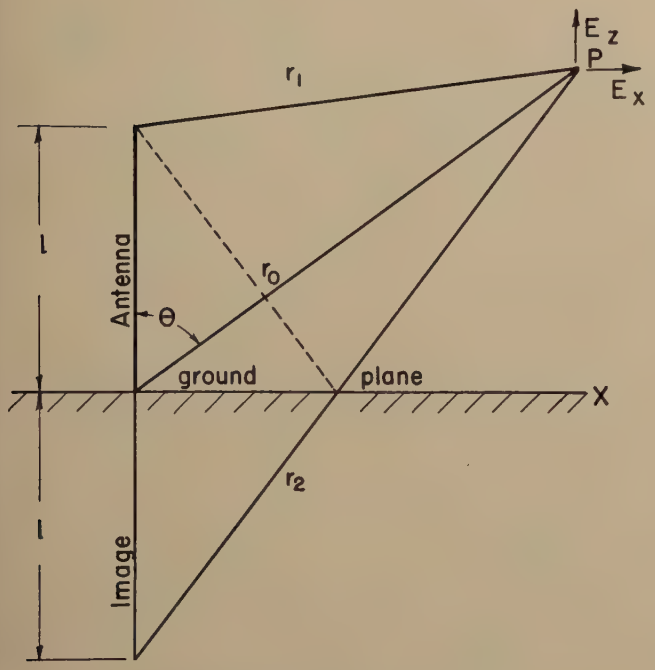


Fig. 12—Geometry of equation (6).

antenna with a sinusoidal distribution can be simulated by two point sources, i.e., a pipe with only two holes in it. (For a quarter-wave antenna, since $\cos kl = 0$, only one hole is needed.)

At a large distance the θ component of E can be found by dividing E_z by $\sin\theta$. At high angles the three components of (6) tend to neutralize each other and completely cancel along the line of the antenna. If two point sources are used their magnitude must be accurately adjusted so as to give complete interference along the line of the antenna. If this adjustment is not accurate, division by $\sin\theta$ would give an infinite value directly above the antenna. Therefore this method is not so accurate for high-angle radiation as the antenna using a line distribution of holes. However, it is very useful at lower angles at short distances and where only the component parallel to z is desired. Such an application will be made to obtain the mutual impedance of antennas.

Mutual Impedance of Two Antennas

Pistolkors,[2] Brown[3] and others have shown that the mutual impedance of two antennas may be computed by integrating along the line of a second antenna the value

$$P = -\int_0^l E_{z12} I_{z1} \cos\Phi dz \qquad (7)$$

where E_{z12} is the voltage induced per unit length in antenna 1 due to current in antenna 2, I_{z1} is the current in antenna 1, and ϕ is the phase difference.

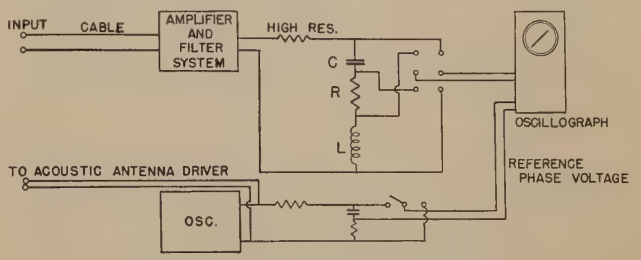


Fig. 13—Phase-measuring circuit.

Pistolkors only computed the mutual impedance of half-wave antennas for definite separations while Brown has indicated the general method for antennas of any height and separation. Brown stated that the simplest method of obtaining the value of the integral is by graphical integration. The acoustic model can perform this integration simply. The first antenna is simulated by radiation from two holes. These two holes must be separated by the length of the antenna, and have a pressure ratio of $-1:\cos k\,l$. For antennas shorter than a quarter wavelength, $\cos k\,l$ is positive and the pressures must be opposite in phase, while for antennas longer than a quarter wavelength, $\cos k\,l$ is negative and the pressures must be in phase. The acoustic pickup is moved along the imaginary line corresponding to the second antenna and the phase relation between the pressure and the reference vector

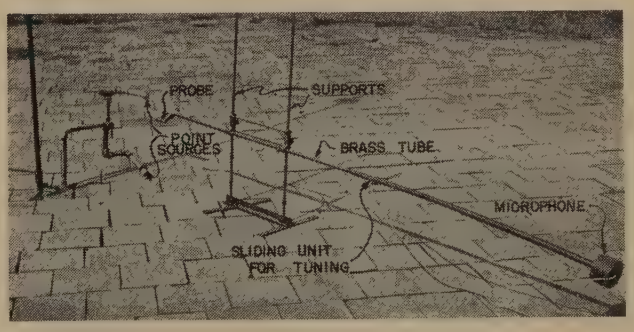


Fig. 14—Physical setup for measuring mutual impedance.

corresponding to the current at the base is measured by the circuit of Fig. 13. The acoustic pickup used is a type of stethoscope. The stethoscope tube is made equal in length to an odd multiple of $\lambda/4$. This increases

[2] A. A. Pistolkors, "The radiation resistance of beam antennas," Proc. I.R.E. vol. 17, pp. 562–579; March, 1929.

[3] G. H. Brown, "Directional antennas," Proc. I.R.E., vol. 25, pp. 78–145; January, 1937.

the sensitivity at the open end and also makes it insensitive to sound picked up through the walls of the tube. Fig. 14 is a photograph and diagram of the two-hole antenna and the stethoscope whose open end is moved along the line of the second antenna.

Some mutual impedance measurements obtained are shown in Figs. 15 to 19.

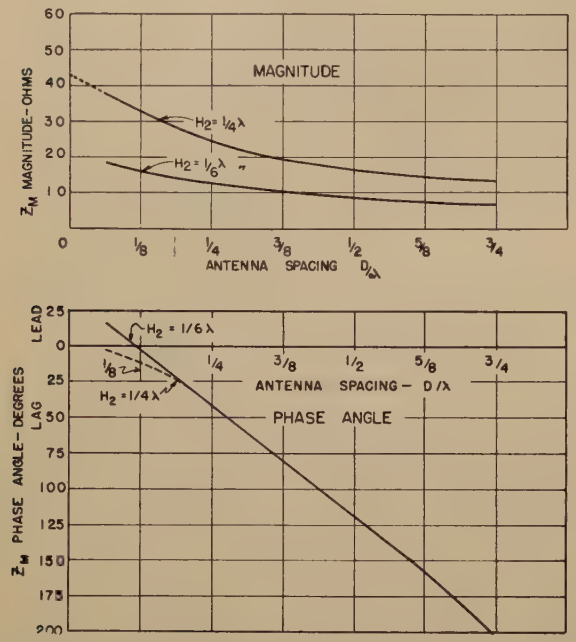


Fig. 15—Mutual impedance where height of one antenna is $\frac{1}{4}\lambda$.

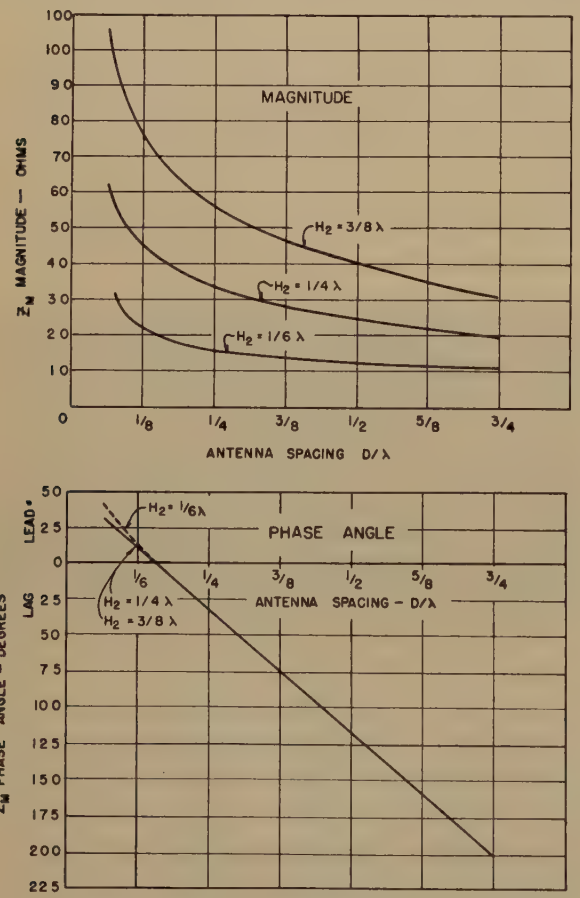


Fig. 16—Mutual impedance where height of one antenna is $\frac{3}{8}\lambda$.

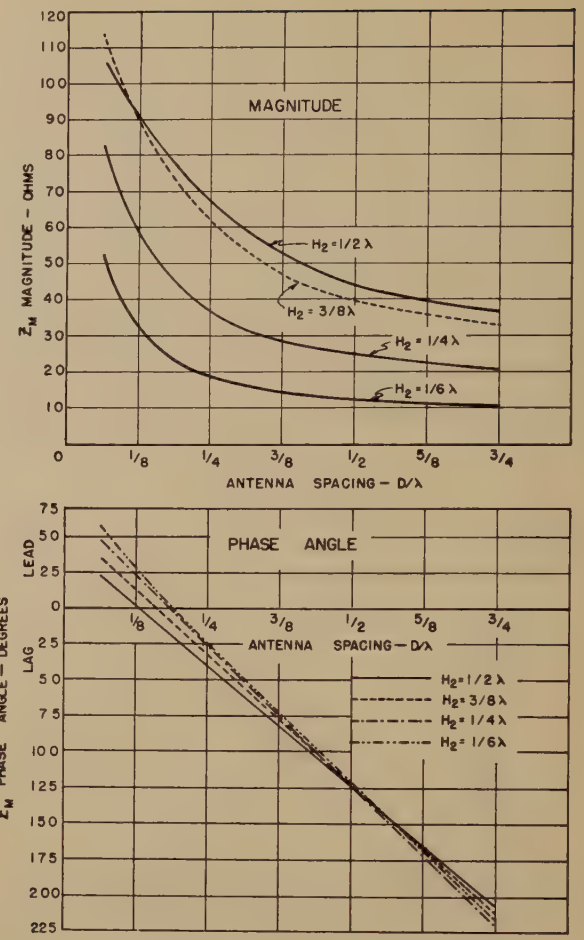


Fig. 17—Mutual impedance where height of one antenna is $\frac{1}{2}\lambda$.

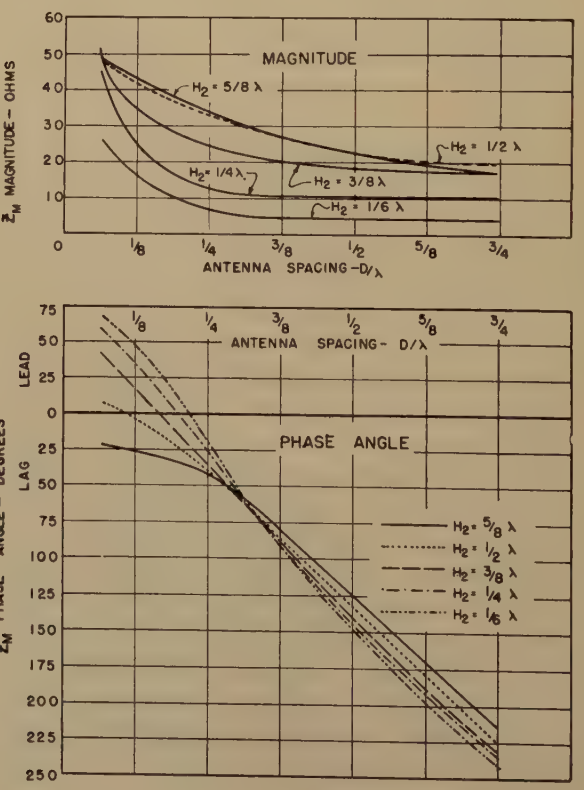


Fig. 18—Mutual impedance where height of one antenna is $\frac{5}{8}\lambda$

This method can also be used to determine the radiation resistance of an antenna. The two-hole source is imperative for this application as the pressure along the multihole antenna does not correspond to the actual E_z close to or along the antenna, while the two-hole method gives exact results for all distances.

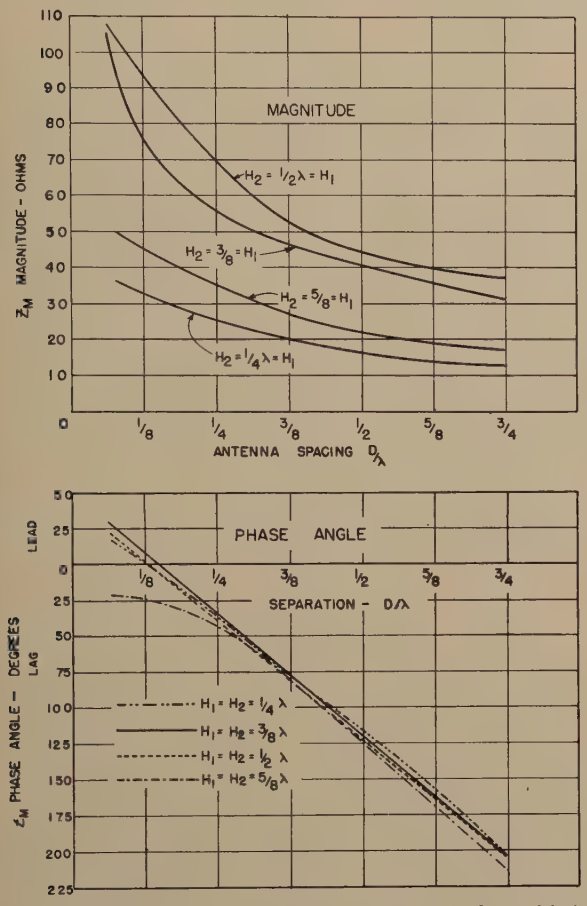

MAGNITUDE
$H_2 = 1/2\lambda = H_1$
$H_2 = 3/8 = H_1$
$H_2 = 5/8 = H_1$
$H_2 = 1/4\lambda = H_1$
Z_M MAGNITUDE - OHMS
ANTENNA SPACING D/λ
PHASE ANGLE
SEPARATION - D/λ
$H_1 = H_2 = 1/4\lambda$
$H_1 = H_2 = 3/8\lambda$
$H_1 = H_2 = 1/2\lambda$
$H_1 = H_2 = 5/8\lambda$
$\angle_M$ PHASE ANGLE - DEGREES LEAD LAG

Fig. 19—Mutual impedance where antennas are of equal height.

An alternative method of determining the radiation resistance of an antenna is to obtain the polar field distribution and then integrate the Poynting vector.

Directional Antennas

The principal utility of the acoustic models would appear to lie in the determination of the field distribution about a directional array. For such a directional array the same types of antennas and measuring equipment which have been discussed would be employed. Two or more acoustic antennas would be used and provision must be made for the control of the phase and magnitude of the excitation of each antenna.

The simplest control of phase of an acoustic system would be obtained by the use of trombone-slide arrangements in the feeding tubes. However, it must be recognized that in a transmission line, either electrical or acoustic, the phase of the excitation of the terminating impedance does not vary directly with the length of the line unless the line is terminated in its characteristic impedance, i.e., unless a wave is traveling in only one direction along the line. For instance if there is a perfect standing wave on a line, the phase of the pressure at the end does not change with the length of the line, but instead the magnitude of the pressure would vary and the phase would stay constant until it reversed when the pressure passes through zero at certain critical lengths.

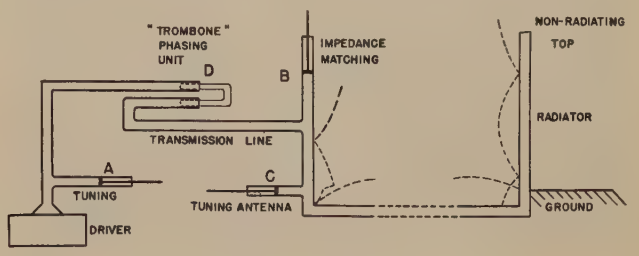


Fig. 20—Acoustic antenna-matching unit.

In the excitation of a multiple-antenna system, it is desirable that one control shall vary the magnitude and another control the phase of the pressure in each antenna. The arrangement corresponding to Fig. 20 was used. The termination of the transmission line is obtained by a method similar to the variation of the taps on an electrical line. It is not feasible to move the tap along a pipe containing a standing-wave system, but the same effect can be obtained by sliding the standing wave along a stationary tap. This is done in the system of Fig. 20 by the two plungers B and C marked "Impedance Matching" and "Antenna Tuning." When the proper adjustment is made, standing waves will be eliminated on the pipe marked "Transmission Line" and the phase shift in the line will then be directly controlled by the "Trombone" phasing unit without affecting the magnitude of the output. The

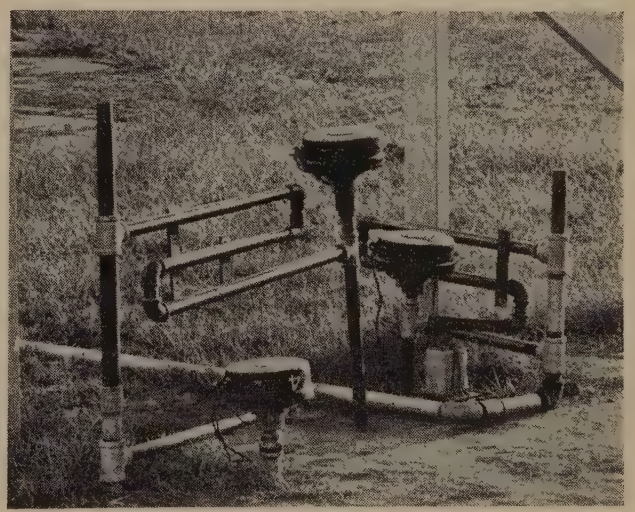


Fig. 21—Photograph of matching units for 3-element array.

magnitude can in turn be independently controlled by adjusting the current in the "Driver" loud-speaker unit.

When several elements of an antenna array are to be excited, there must be a phasing unit for all but one driver. Separate drivers are used for each element.

Fig. 21 shows 3 drivers and 2 phasing units for a 3-element array.

Fig. 22—Model of WLW directional antenna.

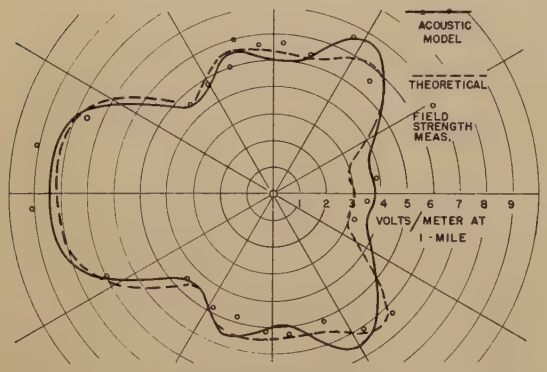


Fig. 23—Field patterns for the WLW directional antenna and its acoustic model.

The phases are adjusted with reference to a common source by the circuit of Fig. 13. A common oscillator supplies all units. Each antenna is in turn energized alone and its phase adjusted with reference to the oscillator source. Due to the lack of appreciable mutual impedance between the elements these relative phase relations will be maintained when all elements are excited simultaneously.

As a test example of the use of a 3-element array, information was obtained from G. F. Leydorf on the design and field surveys of the WLW directional antenna. A model was set up and is shown in Fig. 22. Fig. 23 shows the calculated field pattern, the measured field strengths on the actual radiator, and the measured field on the model, all along the horizon. Fig. 24 shows measurements made on the model at higher angles, and gives the polar pattern which cannot be obtained conveniently from the actual antenna.

The acoustic models have also been applied to non-parallel-antenna arrays, wave antennas, and the construction of acoustic impedances. A more complete treatment of special cases including the design of the unidirectional microphones will be available immediately as a bulletin[4] which may be obtained from The Ohio State University Engineering Experiment Station.

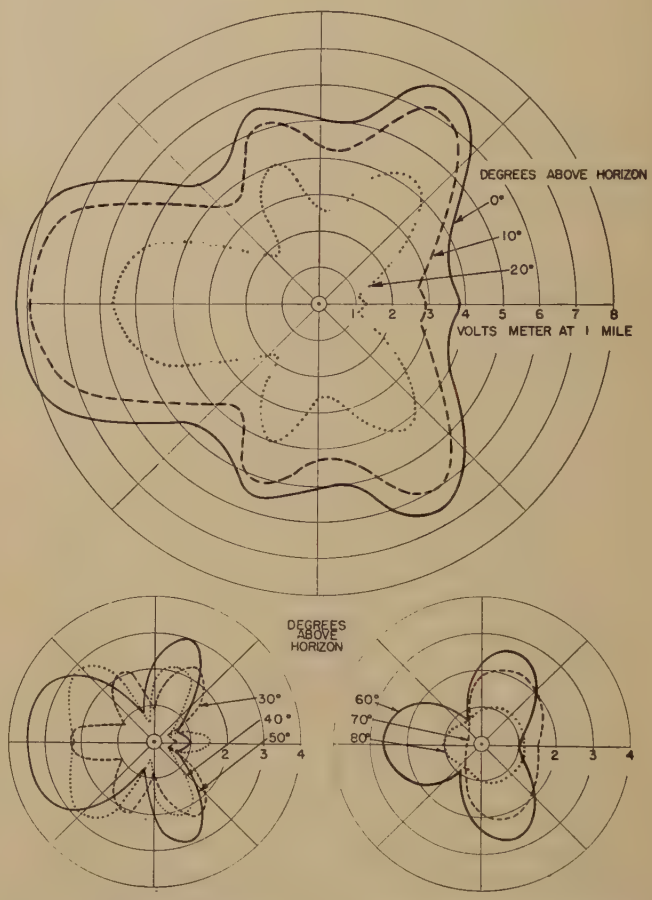


Fig. 24—High-angle fields of WLW directional antenna obtained from the acoustic model.

Conclusion

It can be pointed out that the acoustic models can (a) check the field patterns for special current distributions on single antennas, (b) determine the mutual impedance between two antennas of any character, and (c) determine the field pattern of antenna arrays with individual control of the many independent variables which are possible in such arrays.

[4] E. C. Jordan, "Acoustic Models of Radio Antennas," Bulletin 108, published by The Ohio State University Engineering Experiment Station, Columbus, Ohio.

Oscilloscope Patterns of Damped Vibrations of Quartz Plates and Q Measurements with Damped Vibrations*

H. A. BROWN†, MEMBER, I.R.E.

Summary—*The first part of the paper describes a method of obtaining a picture of the damped vibrations of a quartz-plate resonator which occur when the excitation is removed by short-circuiting the electrodes between which the quartz plate lies. The build-up of vibrations after the short-circuit is removed is also shown. The quartz plate is short-circuited by means of a rotating commutator, and the resulting damped short-circuit current is caused to give a stationary damped-wave pattern on the cathode-ray-oscilloscope screen. Photographs of the patterns are easily obtained.*

Some interesting effects, such as variation of electrode air gap and pressure, are observed, and a few examples are shown.

Since the damping or the Q of a circuit may be readily determined from the formula $S=1/n \log A_1/(A_{n+1})$ applied to n decaying amplitudes from A_1 to A_{n+1} the logarithmic decrement is readily found. The Q is also readily found since it is related to the reciprocal of δ. Amplitudes A_1 and A_{n+1} may be measured on the screen or photograph of the pattern for determination of δ or Q, but a more satisfactory method of measurement of these quantities was developed, incorporating a form of the commonly used momentary contactor, such as is sometimes used in the determination of wave form of an alternator. The speed of the contactor provides a ready determination of the number of vibrations from A_1 to A_{n+1}. A working formula [equation (3)] is developed for rapid measurements of Q of the quartz plate. Some data and characteristics of quartz-plate Q are given as a matter of interest to the reader.

Introduction

SOME five years ago the damped vibrations of a piezoelectric quartz resonator were photographed by Van Dyke.[1] By means of a writing cathode-ray oscillograph, having the photographic plate enclosed in the evacuated chamber, he obtained records of the damped currents which flowed when the resonator electrodes were short-circuited through a low impedance. The records were made by deflecting the cathode-ray beam uniformly along the time base, while the potentials appearing across the short-circuiting impedance produced deflections proportional to the damped currents and, hence, to the mechanical vibrations in the quartz.

Since that time the author of this paper has also become interested in the problem, especially as a means of obtaining stationary patterns on the screen of a cathode-ray oscilloscope of the damped currents which would flow for a brief short-circuit interval. A number of devices, such as impulse generators, electronic switches, etc., were tried without success. A steep wave-front impulse of voltage will easily induce a current in an elementary electric circuit which will subsequently die out exponentially. But in a quartz plate this cannot be done easily, because the inertia of the quartz is very great and it requires a certain interval of excitation to achieve appreciable amplitudes of vibration in the quartz. Therefore, it is necessary to apply an alternating voltage at the proper frequency to the electrodes applied to the quartz-plate resonator until the current builds up to a required value, and to follow this by the short circuit through a low impedance. Two-way discharge tube circuits were tried out as electronic switches to short-circuit the quartz-plate electrodes without success. It seems quite reasonable, however, that a suitable electronic switch may yet be developed.

* Decimal classification: 537.65×R240. Original manuscript received by the Institute, June 25, 1940; revised manuscript received, February 13, 1941.

† Department of Electrical Engineering. University of Illinois, Urbana, Ill.

[1] K. S. Van Dyke, "A determination of some of the properties of the piezo-electric quartz resonator," PROC. I.R.E., vol. 23, pp. 386–392; April, 1935.

Brush Contactor and Test Circuit

A rotary mechanical brush-contact short-circuiting switch was finally developed which gives the desired result quite satisfactorily. This device is illustrated in Fig. 1. At the top of the figure is shown the rotary

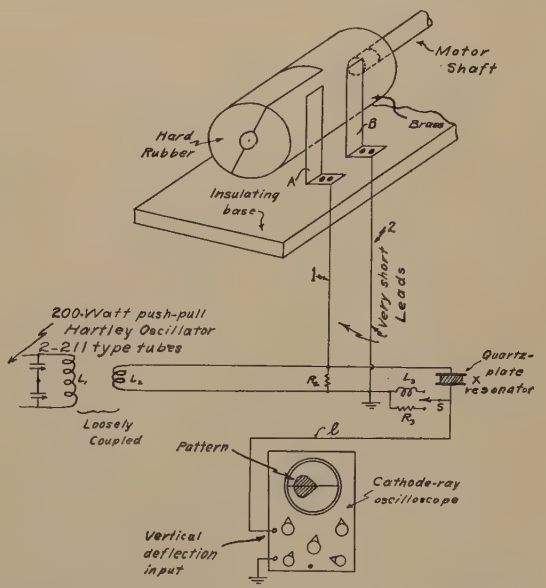


Fig. 1—Short-circuiting commutator and test circuit for obtaining damped-vibration patterns.

switch mechanism, and below, the remainder of the test circuit and apparatus. A piece of cylindrical brass stock first is bored to fit onto a motor shaft, and afterwards turned down to about 1½-inch outside diameter on a mandrel. A semicylindrical portion is then cut away and a piece of hard rubber of a little larger radius is put in place of the removed portion, and secured with screws and glyptal cement. The piece is again turned so that the hard-rubber section has the same diameter and smoothness as the brass portion. The resulting form is shown in the figure. Brush A makes

contact with the brass portion during a half revolution of the rotating cylinder, and the rubber portion opens the contact during the remaining half revolution. Brush B makes contact with the brass portion *only*, as shown by its position.

The quartz-plate resonator is excited into vibration by varying the frequency of a 200-watt oscillator whose tank inductance L_1 is loosely coupled to the reso-

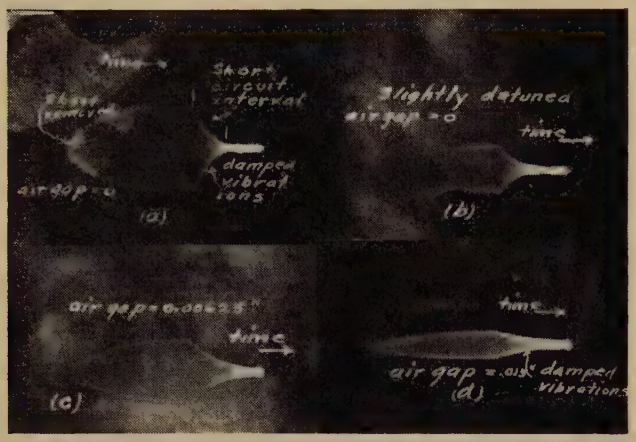


Fig. 2—Oscilloscope patterns of growth and decay of thickness vibrations in a quartz-plate resonator for varying air gap.

nator-circuit pickup coil L_2, with switch S thrown so as to include either L_3 or R_3 in the circuit. With the rotary switch stationary and the rubber portion in contact with brush A, the resonator is not short-circuited; and a fixed amplitude current flows in the resonator when the oscillator frequency is properly adjusted. At this point a solid light rectangular pattern appears on the oscilloscope screen, as the resonator current flowing through L_3 produces a potential drop across this inductance (about 100 microhenrys) to give ample deflection on a 3-inch cathode-ray tube. The pattern is of the solid variety because the sweep frequency is very low compared to the radio-frequency voltage applied to the vertical deflection plates of the oscilloscope cathode-ray tube. The oscilloscope time-deflection generator used was of conventional form, provided with a radio-frequency amplifier-type pentode used in series with the condenser circuit to obtain constant charging rates. The discharge tube was a gaseous triode. The deflection forms an excellent inertia-free indicator of series resonance in the resonator circuit. The motor driving the rotary switch is then started, and when the resonator applied voltage is short-circuited by contact made by brush A the resonator current dies away exponentially, as does the deflection amplitude on the oscilloscope screen. When the brush A contact is opened, current and vibrations again build up in the resonator and hence on the screen. The oscilloscope time-base deflection rate (sweep frequency) is then adjusted to obtain a stationary pattern.

Damped-Vibration Patterns

Typical vibration patterns are shown in Fig. 2. Pattern (a) of the figure was obtained for a 1500-kilocycle X-cut quartz plate with zero air gap between the upper quartz-plate surface and its adjacent electrode, but no pressure was applied. The intervals during which the crystal resonator circuit was short-circuited and the current had completely died out shows clearly, as well as the interval during which resonator current and vibration built up after the short circuit was removed by the opening of brush A contact.[2] The effect of very slight detuning of the exciting frequency is shown at (b) of the figure. In oscillogram (c) an air gap of 0.00625 inch was provided between the upper resonator surface and the electrode, and the lowered damping due to the air gap is apparent. In oscillogram (d) the air gap was 0.015 inch, and shows very low damping with a correspondingly higher value of Q. It is well known that a change in the air-gap length changes the resonant frequency, and in this test it was necessary to retune the oscillator to resonance with the quartz plate each time the air gap was changed. Fig. 3, records (a) and (b), show the effect of applying pressure to the metallic plates of a resonator. The shape of the pattern and steep slope during the short-circuit interval show the greatly reduced Q and higher damping. The increased value of the equivalent resistance is evidenced by the lowered amplitude of deflection. The patterns will effectively show irregularities, such as the effect of slight detuning, as was pointed out, or the

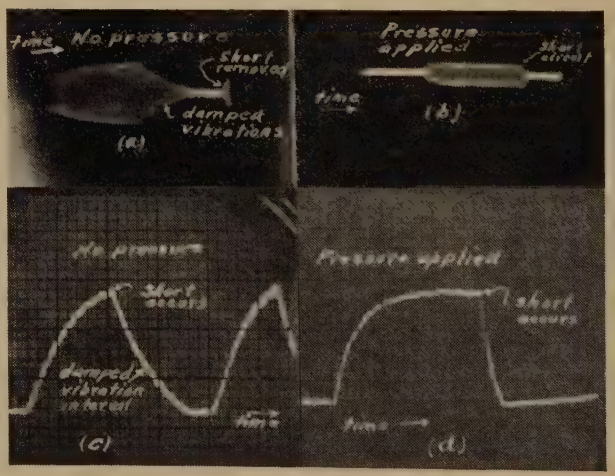


Fig. 3—Oscilloscope patterns of growth and decay of thickness vibrations in a quartz-plate resonator for varying pressure applied to the metallic electrodes.

effect of using rigidly held electrodes, causing increased damping. It has been previously reported that damped vibrations of a low-frequency quartz bar (about 37

[2] It may be mentioned here that various types of short-circuiting contact devices are possible, but it was felt that the precision regularity obtained with the type described was desirable to obtain as stationary a pattern as possible. The pattern flickers, of course, but does not shift back and forth, as it would if the open- and short-circuit intervals fluctuated in a random manner.

kilocycles) were found to last as long as 6 seconds after the exciting electromotive force was removed.[1]

In higher-frequency quartz bars and plates the damped vibrations will not exist over such a large interval of time, but will occur throughout a comparable number of cycles. This is confirmed by measurements of Q, as later described. In the case of the quartz-plate resonator for which oscillogram (a) of Fig. 2 was obtained damped vibrations persisted for a time interval of some 2000 cycles ($1.5/10^3$ seconds) for reduction in amplitude of 60 per cent. For the cases of increased air gaps a greater time is required to decrease in the same proportion.

It is also quite convenient to obtain single-line traces of the build-up and decay of vibrations and current by connecting lead 1 in Fig. 1 to a linear diode detector, the other terminal of which is grounded. The rectified diode detector output voltage is then supplied to the voltage-amplifier input of a commercial-type cathode-ray oscilloscope, incorporating input-voltage amplification. It is hardly necessary to mention that the amplifier connected between the diode detector and the oscilloscope must have high-grade transient response and adequately low frequency cutoff. Oscillogram (c) of Fig. 3 shows the build-up and decay of vibration amplitude for zero pressure on a quartz plate, and (d) shows the same when pressure is applied to the electrodes in contact with the surfaces.

Damping and Q Measurements from Damped-Vibration Amplitudes

The measurement of damping and Q of the quartz resonator may be made by measuring two amplitudes a known number of cycles apart during the damped vibration period. If these amplitudes are A_1 and A_{n+1} and the number of damped vibrations between is n, the logarithmic decrement is

$$\delta = \frac{1}{n}\log_\epsilon \frac{A_1}{A_{n+1}} = \frac{R}{2fL} \qquad (1)$$

where R and L would be the resistance and inductance, respectively, of a damped oscillation in a closed RLC circuit. The mechanical vibrations are assumed to be damped exactly like the currents and potentials in the resonator circuit. The Q of a circuit being defined as $2\pi fL/R$, (1) is readily rearranged to give

$$Q = \frac{2\pi fL}{R} = \frac{\pi n}{\log_\epsilon \dfrac{A_1}{A_{n+1}}}. \qquad (2)$$

The well-known quartz-resonator equivalent circuit is shown in Fig. 4, and the damping or Q, as measured, is really for the entire circuit including L_3 or R_3. When the air gap is very small C_g is sufficiently large so that it may be assumed that the rotary short-circuiting switch (Fig. 1) practically short-circuits C_0 through L_3 or R_3, this latter being very small compared to the reactance of C_0. The damped oscillation, then, takes place in the equivalent quartz circuit RLC. The ideal conditions are attained where the electrodes are formed by plating a metallic film on the surfaces of the quartz plate. If the air gap is considerable the measured Q would not be that of the quartz alone, but would represent that of the rather complex total circuit. The fact that the damping of the decaying current is less when the air gap is increased, as shown in Fig. 2, would indicate that the quartz plate with an air gap between it and one electrode is more desirable for frequency stability in a piezoelectric oscillator, than when the rigid metallic electrodes are in contact with the quartz-plate surfaces. It would be instructive to compare the values of Q of quartz plates as measured with plated

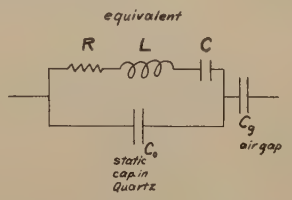


Fig. 4—Equivalent circuit of a piezoelectric quartz plate between metallic electrodes.

electrodes and the values obtained with rigid metallic electrodes in contact with the quartz-plate surfaces.

Measurements of amplitude may be made on the photographic records of Figs. 2 and 3, and after determining the time interval between the selected points for such measurements the Q may be calculated. The interval may be found from a measurement of the sweep frequency of the time-base deflection of the cathode-ray beam. This requires for sufficient facility a variable known-frequency electromotive force which may be impressed on the oscilloscope as a timing wave. The method of measurement just described has two disadvantages. First, the pattern flickers in intensity and it is difficult to keep it from moving slowly, unless some of the input is rectified, amplified, and supplied as a synchronizing voltage. Even then, measurement is not highly accurate because of a somewhat indistinct pattern outline. The pattern is definitely outlined, but the contrast is low, because the cathode-ray-tube screen surface emitted some light around the pattern outline. A better tube may have improved the contrast between the pattern and the background. The pattern may be photographed and the negative will give a fairly sharp outline as is shown in Figs. 2 and 3. If a $3\frac{1}{4}$-$\times 4\frac{1}{4}$-inch or 4-$\times$5-inch camera with a long bellows extension and focusing ground glass be used, full-sized pictures may be obtained at $f/4.5$ lens speed and 0.1- to 0.05-second shutter speed and medium-fast cut films or plates. The pattern may thus be photographed while the sweep frequency is being adjusted until the pattern is stationary. This was done in Figs. 2 and 3.

The photographic method is slow and expensive, hence a method was developed which combined better

precision with greater speed of measurement. The apparatus and test-circuit diagrams are shown in Fig. 5. A rotary contactor composed of a hard-rubber disk, having a brass lug C imbedded in its outer edge, is secured to the end of the rotary short-circuiting switch of Fig. 1. A flat brass ring R is secured to the hard-rubber disk and electrically connected to C with a wire. Brush B_1 makes momentary contact with C and brush B_2 makes continuous contact with R. B_1 has a pointer attached to its mounting support and may be rotated through 180 degrees, the pointer passing over a circular scale of 100 divisions. The commutator and contactor turn clockwise and in the position shown in

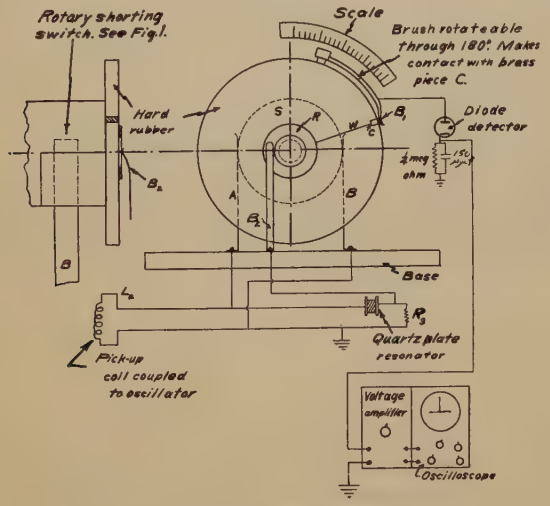


Fig. 5—Contactor and test circuit for damping and Q measurement.

Fig. 5 the quartz resonator is just short-circuited through resistance R_3 of a few hundred ohms. Brush B_1 may be adjusted in position so that a short-time contact is made with C at any time during the dying out of resonator current and vibrations. The short-time contact connects the linear diode detector momentarily across R_3 and a mean rectified impulse voltage is impressed on the input terminals of the voltage amplifier incorporated in the cathode-ray oscilloscope. A vertical-line deflection on the oscilloscope is thus obtained when no horizontal-deflecting electromotive force is applied. The vertical amplitude is readily measured. With the aid of the voltage amplifier full $2\frac{1}{2}$-inch deflections are readily obtained on a 5-inch cathode-ray tube. To make a measurement, brush B_1 is rotated clockwise to a position which gives a deflection on the oscilloscope which is lightly less than that obtained when B_1 is placed so that short circuit has not taken place—position of maximum deflection. This adjustment gives a deflection due to damped current amplitude just after the short circuit occurs, and the position of the pointer on the scale is noted. Then B_1 is moved clockwise to a new position until a deflection is obtained which is $\frac{1}{2}$ or $\frac{1}{3}$ of the first deflection and the deflection as well as the position of the pointer is read. The speed of the rotary switch and contactor driving motor is known, from which the time interval between the two deflections is easily calculated with the aid of the pointer scale readings on the scale of 100 divisions for 180-degree rotation.

The formula for the Q of the resonator is derived as follows: Let A_1 be the larger oscilloscope deflection obtained at a pointer position of g_1 on the graduated 100-division scale, and let A_2 be the smaller deflection obtained when the contactor and pointer are moved to a position of less amplitude and scale reading g_2 during the damped-vibration interval of the short-circuited resonator. The time for $\frac{1}{2}$ revolution of the switch and contactor is $60/2S$ where S is the speed of the motor in revolutions per minute, and in this half revolution the contactor passes over 100 divisions of the pointer scale. The interval between positions g_1 and g_2 is

$$\Delta t = \frac{3(g_2 - g_1)}{10S} \text{ second.}$$

The number of vibrations and the number of cycles of damped current in the quartz resonator in the time Δt is

$$n = \frac{3f(g_2 - g_1)}{10S}$$

where f is the resonant frequency and oscillator frequency. Substituting in (2) the value of the Q of the resonator gives

$$Q = \frac{0.410f(g_2 - g_1)}{S \log_{10} \frac{A_1}{A_2}} \tag{3}$$

f = quartz-plate series-resonant frequency (also exciting frequency in the test)

g_1 = scale reading for deflection A_1, a little less than maximum obtainable

g_2 = scale reading for deflection A_2 equal to $\frac{1}{2}$ or $\frac{1}{3}$ of A_1

S = speed of commutator and contactor driving motor in revolutions per minute.

To illustrate the use of (3) and the method, some measurements were made on a few quartz plates which were available. Table I shows that the Q of quartz plates of varying thicknesses varies in a somewhat random manner, and shows especially that as the frequency increases, or the plate thickness decreases, the Q *does not decrease*. This finding is also reported by Mason,[3] who used an entirely different method. The data of Table I were obtained using an upper electrode of thin brass in light contact with the quartz-plate surface. Electrodes formed by silver or platinum plating on the quartz surface would of course give values more nearly correct for the quartz proper, as were used by Mason.

[3] W. P. Mason (Discussion of "Transmission lines at very high radio frequencies," by L. E. Reukema), *Trans. A.I.E.E.* (*Elec. Eng.*, February, 1938), vol. 57, pp. 105–107; 1938.

TABLE I

Thickness Vibration Frequency	Type of Cut	Q
Kilocycles	Cut	
610	Y	32,000
1500	Y	29,500
1660	X	15,000
1879	AT	24,000
2650	X	40,000
3680	AT	28,000

The effect of the increasing pressure on an X-cut quartz plate as illustrated by the patterns of Fig. 3 is shown by curve (b) of Fig. 6, the Q decreasing when the electrode pressure is increased. This is the 1660-kilocycle quartz plate shown in Table I. In the same figure curve (a) shows the effect of varying the length of the air gap between the upper electrode and the upper surface of a 1-inch square X-cut quartz plate. In both quartz plates for which Fig. 6 is obtained thickness vibrations very probably take place because of the relation of the exciting frequency to the thickness of the plate. In general the Q of the entire resonator increases with air-gap length, except in the critical region of point P where the air gap is probably near a half wavelength of the supersonic stationary air wave existing in the air gap.[4] The value of Q shown for the 0.05-inch air gap is questionable. It may be that with such large air gaps some sort of coupled damped-oscillation phenomenon takes place.

Conclusions

1. The rotary short-circuiting switch or commutator makes it convenient to observe the rate at which vibrations in quartz decay when the driving electromotive force of a quartz resonator is removed, if the dying-out phenomenon is repeatable with sufficient rapidity to be observable on the oscilloscope screen. It is suitable for the higher-frequency quartz plates where the decaying interval is a small fraction of a second. The apparatus is a useful tool or means of quickly observing the effect of changing some factor in the operation of a quartz-plate resonator, such as pressure change, air-gap length variation, and probably a number of others.

2. From the observed or photographed patterns, amplitudes may be measured at two different intervals during the decaying vibration period, and the damping or Q measured. A method is worked out, using a short-time contactor in conjunction with the short-circuiting commutator. This method is capable of rapid manipulation and is convenient when a large amount of data is to be obtained.

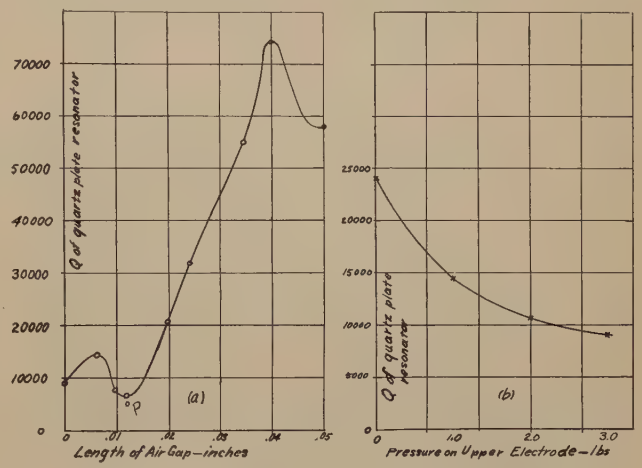


Fig. 6—Variation of Q of quartz-plate resonator with air gap and pressure on the upper electrode.

Acknowledgment

The author acknowledges and appreciates the assistance of Mr. P. W. Ryburn and Mr. S. S. Brar. Mr. Ryburn, a former senior student in electrical engineering, made several attempts to use transient observing methods with impulse voltages and electronic switching means, which did not prove applicable. Mr. Brar, a former graduate student in electrical engineering, further investigated the possibilities of electronic short-circuiting and assisted in setting up the first circuit using the mechanical method of short-circuiting the resonator.

[4] D. W. Dye, "The piezo electric quartz resonator and its equivalent electric circuit," *Proc. Phys. Soc.* (London), vol. 38, pp. 399–458; December–August, 1925–1926.

Large-Signal High-Frequency Electronics of Thermionic Vacuum Tubes*

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Summary—*Electronics equations which can be used to analyze the type of signals involved in class C amplifiers and oscillators are completely worked out. There are four possible modes of operation in a space-charge-limited diode, when a pulsive voltage is applied to it. The mode of operation for a fixed wave form of the signal depends upon one normalization constant which is in direct proportion to the magnitude of the voltage and in inverse proportion to the square of the product of the dimension of the tube and the frequency. The method for calculating the performance of the conventional power tube at high frequencies is given and the ultimate limit of the tube is indicated.*

I. Introduction

THE small-signal theories of ultra-high-frequency electronics have been completely and extensively studied by previous workers including Benham,[1,2] Müller,[3] Llewellyn,[4,5] Grünberg,[6] and others. All such theories are based upon the assumption that a steady space-charge distribution has been established inside a vacuum tube and the high-frequency signal is merely a disturbance small in comparison with the steady component. The mathematical technique is to take advantage of the smallness of the signal so that a rapidly convergent series is obtainable. As remarked by Llewellyn,[4,5] such investigations are entirely inadequate for the case of large signals such as one usually encounters in a power amplifier or oscillator. The complication of the mathematical formulas in the small-signal theories leads one to believe that it would be very difficult to extend the same method to the large-signal analysis. Therefore, Grünberg[7], using a somewhat different approach, made an investigation of the response of a plane diode to a Heaviside unit electromotive force. This is generally recognized as a very good start for the large-signal analysis. However, the analysis is not complete, as it only gives the initial rise of current of a temperature-limited diode and fails to show how the final space-charge condition in the tube is established. Even if the solution be completed, it cannot be applied to the analysis of a practical tube, because an ordinary vacuum tube is generally operated in the space-charge-limited region and the boundary conditions are nonlinear so that it is impossible to apply the superposition theorem as in the linear-network analysis. This paper offers a method which completely describes the high-frequency phenomena inside a vacuum tube when large impulsive voltages and currents are involved. The solution degenerates into the space-charge-limited operation at low frequencies. For the convenience of analysis, we divide a multiple-electrode vacuum tube into several diode sections. Each region between two adjacent electrodes is treated as a diode with the corresponding boundary conditions. In section II, we shall discuss the solution of a plane diode with a cathode of infinite emission capacity, from which the electrons are emitted with zero initial velocity as assumed by Child. This cathode has the property of preventing a negative electric field from developing on its surface. (Negative field is one which accelerates negative-charged electrons in the positive direction from the cathode toward the plate.) That is to say, the cathode will emit enough electrons to keep a zero electrical field on its surface whenever there is a tendency to develop a negative field, but it will emit no electrons whenever a positive field is acting on it. In section III, we shall discuss the case of a plane diode with a periodic injection of electrons of specified charge density and velocity at the first electrode. This simulates the condition in the grid-plate space of a power triode and also the screen-plate space of a power tetrode.

It should be noted that in this investigation the frequencies considered are limited to the moderately high-frequency range because we neglect the time of propagation of electromagnetic waves across the tube space in comparison with the time period of the signal and the time of transit of the electrons. This is not a serious limitation in the investigation of ordinary power tubes as we are principally interested in the first deviation from the low-frequency analysis. With this assumption in mind, we can proceed in either of two ways. We can set the electric field directly equal to the negative of the gradient of the scalar potential as in the "total current method"; or we can use Poisson's equation obtaining an electrostatic mapping as is done in the "image method." Since these two methods are based upon the same assumption they should mutually check each other. In the following analysis, we shall

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NOTE: During the author's stay in the RCA Manufacturing Co., he had the pleasure to learn that Mr. B. Salzberg has carried out an independent research concerning the same subject. We are satisfied in having checked each other's results in certain phases. His results will be published in the near future and space-charge-free cases will be presented in detail. The author thanks him in suggesting better symbols and units during the publication of this paper.

[1] W. E. Benham, "Theory of the internal action of thermionic systems at moderately high frequencies, Part I," *Phil. Mag.*, vol. 5, pp. 641–662; March, 1928.

[2] W. E. Benham, "Theory of the internal action of thermionic systems at moderately high frequencies, Part II," *Phil. Mag.*, vol. 11, pp. 457–517; February, 1931.

[3] J. Müller, "Elektronenschwingungen in Hochvacuum," *Hochfrequenz. und Elektroakustik*, vol. 41, pp. 156–167; May, 1933.

[4] F. B. Llewellyn, "Vacuum tube electronics at ultra-high frequencies," PROC. I.R.E., vol. 21, pp. 1532–1573; November, 1933.

[5] F. B. Llewellyn, "Operation of ultra high frequency vacuum tubes," *Bell Sys. Tech. Jour.*, vol. 14, pp. 632–665; October, 1935.

[6] G. Grünberg, "Zur Theorie der Wirkungsweise von Elektronenröhren bei rasch Veränderlicher Anodenspannung," *Tech. Phys.* (U.S.S.R.), vol. 3, pp. 65–80; January, 1936.

[7] G. Grünberg, "Über den Anfangsstrom, der durch eine Elektronenröhre beim plötzlich anlegen einer Stosspannungen fliesst," *Tech. Phys.* (U.S.S.R.), vol. 3, pp. 101–110; February, 1936.

follow the total-current method but we shall also use the conception of the image method in splitting the total current into two components as follows:

$$I = I_c + I_i, \quad \text{where} \tag{1}$$

I = the total current per unit area flowing through the plane diode

I_c = the capacitive current per unit area resulting from the variation of the voltage

$$= c\frac{dV}{dt}, \quad \text{and}$$

I_i = the induced current[8] resulting from the motion of the electrons

$$= \int_0^d \frac{\rho(\xi)u(\xi)}{d}\,d\xi.$$

In the above equations, V is the applied voltage; ρ and u are the charge density and velocity of electrons as a function of the space and co-ordinate ξ which is the distance from the first electrode; d is the distance between the two electrodes; c is the capacitance per unit area and is equal to ϵ/d.

Only the component of current I_i has real interest in a power tube because the capacitive component is usually separated out as though in an external branch and produces no average power.

In power-tube oscillators and amplifiers, the grid bias varies from zero for class B operation to some high negative voltages for class C operation. At low frequencies, it is evident that the electrons in one cycle are completely removed from the space before the next cycle starts and in the space there will be no intermingling of electrons emitted from the cathode at different instants. This condition will always be true for all frequencies in high-biased class C operation and will only be true for low-biased cases up to a certain frequency. Above this frequency, which depends upon the magnitude of bias, some electrons of one cycle will stay in the space as long as the emitter begins to emit in the next cycle and there will be space charge in the space throughout the whole cycle just like the condition in the space of a class A amplifier. Finally a steady state will be reached in which electrons are only emitted in an interval beginning at a time between $t=0$ and $t=t_0$ in Fig. 1 and ending at a time between $t=t_0$ and $t=2t_0$. The following analysis only includes cases in which electrons in one cycle will be removed either by the plate or the cathode before the next cycle starts. The steady-state operation of low-biased cases at high frequencies will require another set of electronics equations.

[8] Parallel formulas can be derived for tubes of other configurations, such as

I_i = induced current per unit length for a cylindrical diode

$$= \int_a^b \frac{2\pi\rho(r)u(r)}{\log b/a}\,dr$$

I_i = induced current for a spherical diode

$$= \int_a^b \frac{4\pi ab\cdot\rho(r)u(r)}{b-a}\,dr$$

a and b are the inside and outside radii, respectively.

II. A Space-Charge-Limited Diode

In this section we shall study the phenomena inside a plane diode when a voltage pulse such as that shown in Fig. 1 is applied to it. This voltage has a peak value

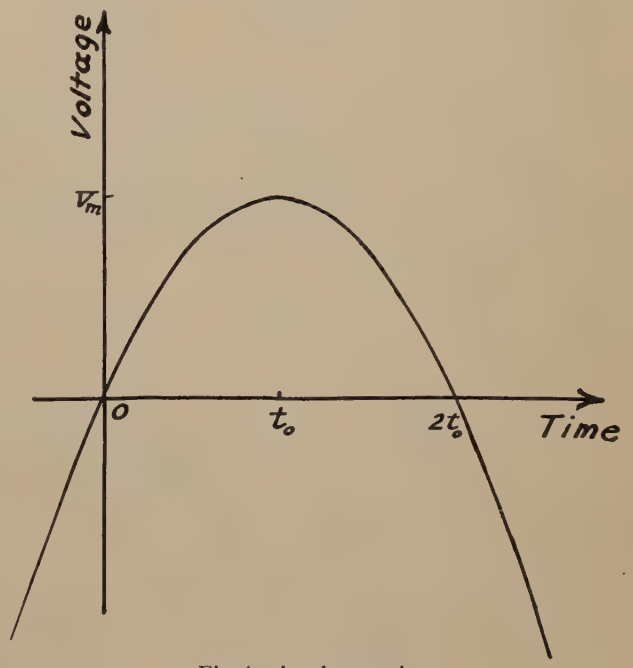


Fig. 1—A voltage pulse.

V_m and is positive during a time $2t_0$. We shall, for convenience of argument, only consider a symmetrical voltage pulse. The following theory can, however, be applied to any pulse provided only that it rises gradually from negative to positive values through a maximum and then decreases again to negative values. It is evident that before the time $t=0$ there are no electrons inside the tube space because an ideal emitter is unable to emit due to the retarding electric field. After the time $t=0$, the voltage begins to go positive and the cathode emits enough electrons to maintain a zero field at its surface. Between $t=t_0$ and $t=2t_0$, the voltage begins to drop and at a certain instant within this interval the emission current drops to zero and remains zero because, in spite of the fact that the plate voltage is still positive, a retarding field develops on the surface of the cathode due to the space charge. This will become apparent in the course of the following analysis. In Fig. 2 we can plot the distance x traveled by different electrons at various times. Each curve is specified by the variable τ which is equal to the time at which the particular electron is emitted. To describe such curves, we indicate several times of interest as follows:

t_1 = the time at which the first electron emitted at the time $t = 0$ reaches the plate.

t_2 = the time at which the emitter stops emitting electrons.

t_3 = the time at which the plate receives the last electrons.

t_4 = the time at which the last electron in the space is removed by the cathode and after which the space becomes empty.

case (d), the reversal of the voltage is so fast that no electron has a chance to touch the plate before it is retarded and turned back to the cathode. As the frequency increases or t_0 decreases, the ratios of t_1, t_3, and t_4 to t_0 increase while t_2/t_0 decreases. We see that the different lines of each diagram of Fig. 2 can be considered as contour lines cut from a surface sheet formed by plotting the value τ against x and t in a three-dimensional co-ordinate system. For each mode of

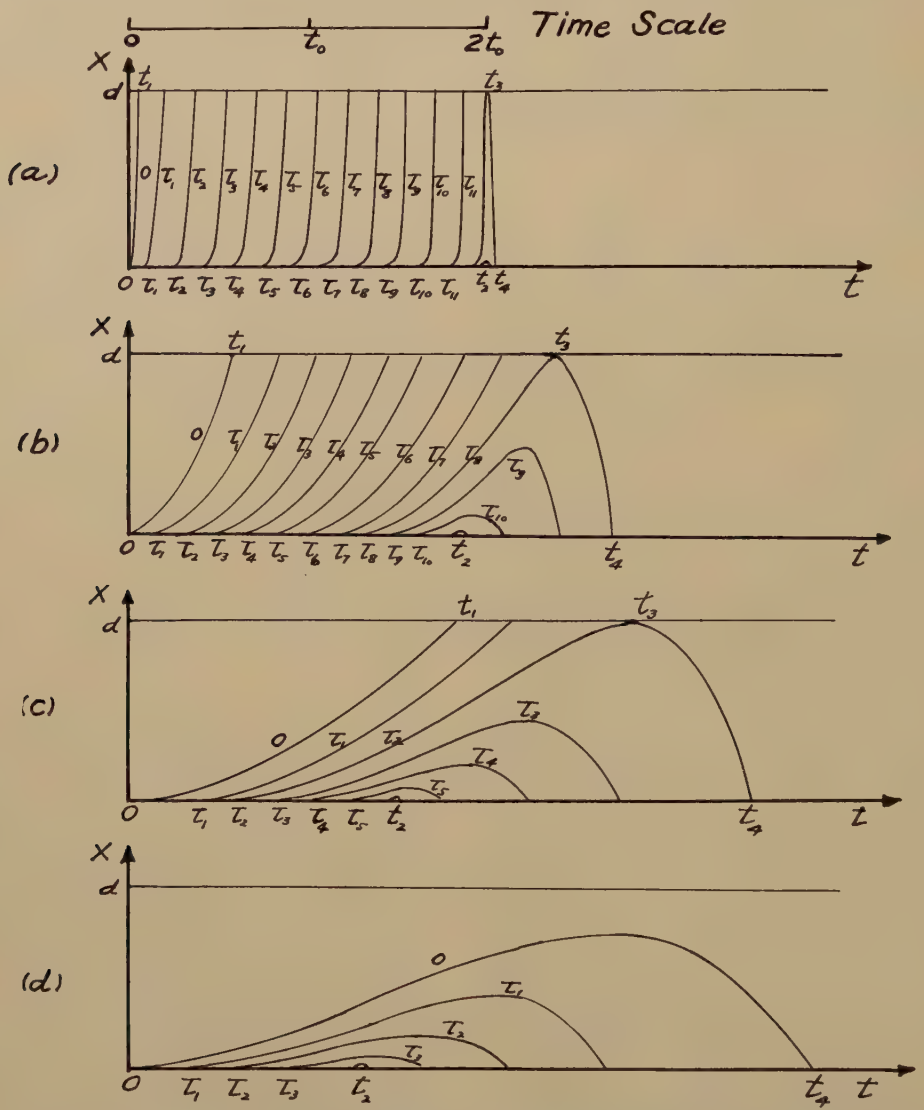


Fig. 2—Space co-ordinate x occupied by different τ electrons at time t, when a voltage pulse such as in Fig. 1 is applied to a plane diode. Four possible modes of operation are represented by four different diagrams with the relative transit time increasing in order from (a) to (d).

The four different diagrams in Fig. 2 represent four different modes of operation which for the convenience of argument will be called cases (a), (b), (c), and (d), respectively. Case (a) represents the quasi-stationary-state analysis in which the transit time is negligibly small. In case (b), the transit time becomes a considerable fraction of the interval $2t_0$. For still longer transit time, we have case (c), which differs from case (b) only by the fact that the cathode stops emitting electrons before the first electron reaches the plate, i.e., $t_2 < t_1$. In

operation, there exists one such surface sheet. Since in a nonstationary-state analysis, every physical quantity is in general a function of both the space co-ordinate x and the time co-ordinate t, through the functional relationship formed by the above surface sheet, every quantity can be expressed equally well as a function of t and τ co-ordinates. To avoid ambiguity, we use the square bracket to denote the functional relation with t and x as independent variables and the round bracket to express the functional relation with t and τ as inde-

pendent variables. Thus a function f can be expressed in either of the alternative ways,

$$= f[t, x] = f(t, \tau)$$

in which x, t, and τ are related to each other by the functional relation for axes transformation

$$x(t, \tau) \quad \text{or} \quad \tau[t, x]$$

for each mode of operation.

In Fig. 3 successive diagrams are drawn for each mode of operation to show the space-charge conditions inside the tube. The crosshatched areas represent the presence of space charge. The arrowhead represents the direction of the moving electrons. With this in mind we can define symbols and notations as follows:

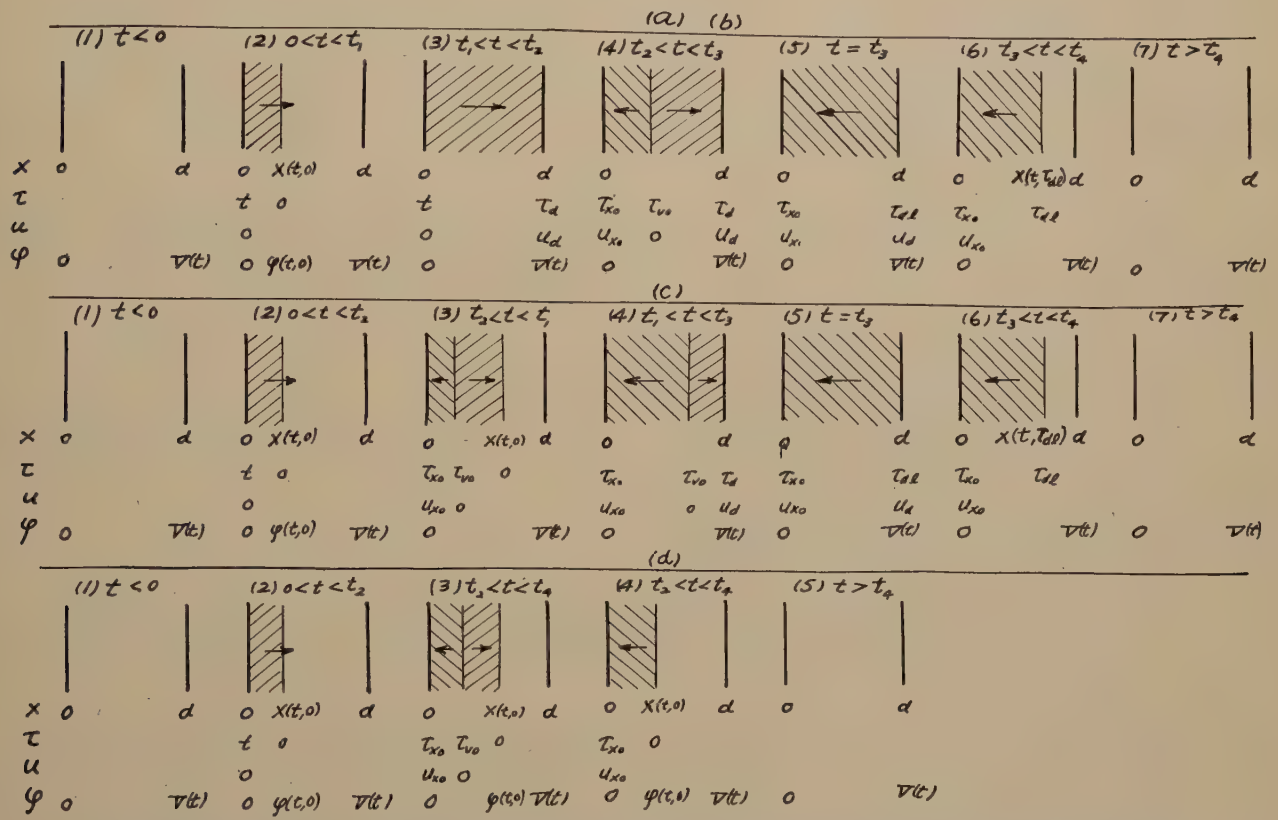


Fig. 3—Space-charge condition in a plane diode in different periods when a voltage pulse is applied. (a) and (b) cases (a) and (b). (c) case (c). (d) case (d). Crosshatched areas indicate the presence of space charge. Arrowhead represents the direction of the velocity of the moving electrons.

Symbols and Definitions

(all quantities in meter-kilogram-second system)

(a) *Independent Variables and Constants*

x = the distance measured from the cathode surface

t = the time as measured from the instant when the voltage is zero and is going from negative to positive

τ = the variable assigned to a particular group of electrons which starts from the cathode at the time $t=\tau$. This group of electrons will be called τ electrons

d = the distance between the cathode and the plate. Hence, $x=0$ at the cathode and $x=d$ at the plate

ϵ = the electric inductive capacitance (in vacuum, $\epsilon=\epsilon_0=1/36\pi\cdot 10^9$)

e = the electric charge of an electron

m = the mass of an electron

c = the capacitance per unit area $=\epsilon/d$

t_0 = the half period of a pulse as in Fig. 1

V_m = the peak voltage of a pulse as in Fig. 1

ω = the angular velocity of the applied voltage

f = the frequency of the applied voltage

$\gamma = t_0 f$

$T = t/t_0 \quad \theta = \omega t$

C = the normalization constant $= 2eV_m t_0^2/md^2 = 2eV_m\gamma^2/md^2f^2$

K = the normalization constant $= 2V_m/2m\omega^2d^2$

$a = e/2md$

(b) *Quantities Which are Functions of Time t only*

$V(t)$ = the external applied voltage between the plate and the cathode

$I(t)$ = the total current per unit electrode area passing from the cathode to the plate through the outside circuit

$I_c(t)$ = the capacitive current component per unit area resulting from the variation of the voltage.

$I_i(t)$ = the induced current component per unit area resulting from the motion of the electrons

The following quantities are mathematically defined:

$$Q(t) = \int_0^t I(\eta)d\eta$$

$$Q'(t) = I(t)$$

$$Q_c(t) = \int_0^t I_c(\eta)d\eta = \epsilon \frac{V(t)}{d}$$

$$Q_c'(t) = I_c(t) = \epsilon \frac{V'(t)}{d}$$

$$Q_i(t) = \int_0^t I_i(\eta)d\eta$$

$$Q_i'(t) = I_i(t).$$

By equation (1), it is evident that

$$Q(t) = Q_c(t) + Q_i(t)$$

$$Q'(t) = Q_c'(t) + Q_i'(t).$$

(c) *Quantities Which Are Functions of Both Time t and Distance x, or Time t and τ Co-ordinate*

$D[t, x]$ or $D(t, \tau)$ = the electric displacement
$E[t, x]$ or $E(t, \tau)$ = the electric field
$\phi[t, x]$ or $\phi(t, \tau)$ = the scalar potential as measured from the cathode potential
$\rho[t, x]$ or $\rho(t, \tau)$ = the average space-charge density of electrons
$u[t, x]$ or $u(t, \tau)$ = the average velocity of electrons
$x(t, \tau)$ = the space co-ordinate of the particular electron at the time t
$\tau[t, x]$ = the particular τ electron which occupies the space co-ordinate x at the time t

(d) *Abbreviations*

$\tau_d = \tau[t, d]$ = the particular τ electron at the plate when time is equal to t
$\tau_{x0} = \tau[t, 0]$ = the particular τ electron at the cathode when time is equal to t
τ_{v0} = the particular τ electron which has zero velocity when time is equal to t
τ_{dl} = a constant, the last τ electron which reaches the plate and also the last electron which returns to the cathode
$u_d = u[t, d] = u(t, \tau_d)$ = the velocity of the electrons at the plate
$\rho_d = \rho[t, d] = \rho(t, \tau_d)$ = the charge density of the electrons at the plate
$u_{x0} = u[t, 0] = u(t, \tau_{x0})$ = the velocity of the electrons at the cathode
$\rho_{x0} = \rho[t, 0] = \rho(t, \tau_{x0})$ = the charge density of the electrons at the cathode
$\Delta t = t - \tau_d$ = the transit time.

Hence, by definition,
at $t = t_3$, $x(t, \tau_{dl}) = d$
at $t = t_4$, $x(t, \tau_{dl}) = 0$.

General Electronics Equations and Boundary Conditions for the Plane Diode

Here we copy down the electronics equations in the current literature with our present notations.

$$\frac{\partial D[t, x]}{\partial x} = -\rho[t, x] \tag{2}$$

$$-Q'(t) = -I(t) = -\rho[t, x]\cdot u[t, x] + \frac{\partial D[t, x]}{\partial t} \tag{3}$$

$$\frac{D[t, x]}{\epsilon} = E[t, x] = -\frac{\partial \phi[t, x]}{\partial x} = -\frac{\dfrac{\partial \phi(t, \tau)}{\partial \tau}}{\dfrac{\partial x(t, \tau)}{\partial \tau}} \tag{4}$$

$$\frac{\partial^2 x(t, \tau)}{\partial t^2} = -\frac{e}{m} E(t, \tau). \tag{5}$$

Substituting (2) into (3) and remembering the Eulerian time-differentiation convention of total differentiation for a moving particle and partial differentiation for fixed space, we have,

$$Q'(t) = -\frac{dD[t, x]}{dt} = -\frac{\partial D(t, \tau)}{\partial t}. \tag{6}$$

We can integrate (4) with respect to τ from $\tau = \tau_{x0}$ to $\tau = \tau$, noticing that $\phi[t, 0] = \phi(t, \tau_{x0}) = 0$ by definition,

$$\phi(t, \tau) = -\int_{\tau_{x0}}^{\tau} E(t, \eta) \frac{\partial x(t, \eta)}{\partial \eta} d\eta. \tag{7}$$

The different boundary conditions are as follows: zero electric field at the cathode during emission, i.e.,

$$E(\tau, \tau) = 0 \tag{8}$$

and zero initial velocity, which gives

$$u(\tau, \tau) = \left.\frac{\partial x(t, \tau)}{\partial t}\right|_{t=\tau} = 0. \tag{9}$$

Specified external applied voltage between the plate and the cathode,

$$\phi[t, d] = \phi(t, \tau_d) = V(t). \tag{10}$$

Also by definition,

$$x(\tau, \tau) = 0. \tag{11}$$

With the above fundamental equations of electronics and boundary conditions, we can derive secondary equations.

Integrating (6) with respect to t in view of (8) and (5), we have

$$\left.\begin{aligned} D(t, \tau) = \epsilon E(t, \tau) &= -Q(t) + Q(\tau) \\ \frac{\partial^2 x(t, \tau)}{\partial t^2} &= \frac{e}{m\epsilon}[Q(t) - Q(\tau)] \end{aligned}\right\}. \tag{12}$$

Integrating again and using (9), we have,

$$u(t,\tau) = \frac{\partial x(t,\tau)}{\partial t} = \frac{e}{m\epsilon}\int_{\tau}^{t} [Q(\eta) - Q(\tau)]d\eta = \frac{e}{m\epsilon}\int_{\tau}^{t} (t-\eta)Q'(\eta)d\eta. \quad (13)$$

Integrating again and using (11), we have,

$$x(t,\tau) = \frac{e}{m\epsilon}\int_{\tau}^{t} d\xi \int_{\tau}^{\xi} (\xi-\eta)Q'(\eta)d\eta. \quad (14)$$

This is a double integral above the $\eta-\xi$ plane which can be converted into a single integral by changing the order of integration with respect to the two variables η and ξ. This then becomes

$$x(t,\tau) = \frac{e}{m\epsilon}\int_{\tau}^{t} \frac{(t-\eta)^2}{2} Q'(\eta)d\eta. \quad (15)$$

When it is differentiated with respect to τ, it becomes,

$$\frac{\partial x(t,\tau)}{\partial \tau} = -\frac{e}{m\epsilon}\frac{(t-\tau)^2}{2} Q'(\tau). \quad (16)$$

By definitions of τ_d, τ_{x0}, and τ_{v0}, we can obtain the following equations from (13) and (15),

$$d = \frac{e}{m\epsilon}\int_{\tau_d}^{t} \frac{(t-\eta)^2}{2} Q'(\eta)d\eta \quad (17)$$

$$0 = \frac{e}{m\epsilon}\int_{\tau_{x0}}^{t} \frac{(t-\eta)^2}{2} Q'(\eta)d\eta \quad (18)$$

$$0 = \frac{e}{m\epsilon}\int_{\tau_{v0}}^{t} (t-\eta)Q'(\eta)d\eta. \quad (19)$$

We can differentiate (17) with respect to t, remembering the definitions of u_d and u_{x0}, and also (13),

$$u_d = \frac{e}{m\epsilon}\frac{(t-\tau_d)^2}{2} Q'(\tau_d)\tau_d'(t) \quad (20)$$

$$u_{x0} = \frac{e}{m\epsilon}\frac{(t-\tau_{x0})^2}{2} Q'(\tau_{x0})\tau_{x0}'(t) \quad (21)$$

where $Q'(\tau_d)$ and $Q'(\tau_{x0})$ are functions of τ_d and τ_{x0}, respectively, which are themselves functions of t.

Substituting (12) and (16) into (7), we obtain

$$\phi(t,\tau) = -\frac{e}{m\epsilon^2}\int_{\tau_{x0}}^{\tau} [Q(t) - Q(\eta)]\frac{(t-\eta)^2}{2} Q'(\eta)d\eta. \quad (22)$$

Multiplying (18) by $Q(t)$ and adding to (22), we derive the following equation by combining integrals:

$$\phi(t,\tau) = \frac{1}{\epsilon}Q(t)x(t,\tau) - \frac{e}{m\epsilon^2}\int_{\tau}^{\tau_{x0}} \frac{(t-\eta)^2}{2} Q(\eta)Q'(\eta)d\eta. \quad (23)$$

Solution for a Pulsive Voltage

Case (a) Quasi-Stationary State Analysis

Child's equation for space-charge-limited operation of a diode in a stationary state is well known. For a plane diode it is

$$I = \frac{4\epsilon}{9}\sqrt{\frac{2e}{m}}\frac{V^{3/2}}{d^2} \quad (24)$$

where I and V are, of course, constant quantities. The static-characteristic-plotting method assumes that V and I can be functions of time in such a way that they are functionally related to each other by (24). We shall show here that this is only approximately correct in the case when the transit time is small. When the transit time is small, the tube will be completely filled in by electrons as long as the voltage is positive and we can always find a τ_d electron at the plate. Let us denote the small quantity $t-\tau_d$ by Δt. Then by (17), (10), and (22) we have

$$d = \frac{e}{m\epsilon}\int_{t-\Delta t}^{t} \frac{(t-\eta)^2}{2} Q'(\eta)d\eta \quad (25)$$

$$V(t) = \phi(t,\tau_d) = \frac{e}{m\epsilon^2}\int_{t-\Delta t}^{t} [Q(t) - Q(\eta)]Q'(\eta)\frac{(t-\eta)^2}{2}d\eta. \quad (26)$$

As Δt is so small, $Q(\eta)$ can be expressed as a rapidly convergent Taylor's series around $Q(t)$ within the integration limits of (25) and (26). When the integration is carried out on this series, we obtain the equations

$$d = \frac{e}{m\epsilon}\left[Q'(t)\frac{(\Delta t)^3}{6} - Q''(t)\frac{(\Delta t)^4}{8} + \cdots\right] \quad (27)$$

$$V(t) = \frac{e}{m\epsilon^2}\left[Q'^2(t)\frac{(\Delta t)^4}{8} - Q'(t)Q''(t)\frac{3(\Delta t)^5}{20} + \cdots\right]. \quad (28)$$

Let us try a first-order solution by neglecting all the higher-order terms. Call this first-order solution $Q_1'(t) = I_1(t)$, such that

$$d = \frac{e}{m\epsilon}Q_1'(t)\frac{(\Delta t)^3}{6} \quad (29)$$

$$V(t) = \frac{e}{m\epsilon^2}Q_1'^2(t)\frac{(\Delta t)^4}{8}. \quad (30)$$

Eliminating Δt from (29) and (30), we have,

$$I_1(t) = Q_1'(t) = \frac{4\epsilon}{9}\sqrt{\frac{2e}{m}}\frac{V^{3/2}(t)}{d^2}.$$

This equation is exactly the same as (24) except that I_1 and V can be functions of time. This can be called the dynamic form of Child's equation, the existence of which substantiates the static-characteristic-plotting methods. In order to obtain the criterion under which the dynamic Child's equation is true, it is necessary to examine the second-order solution. Let us call the

second-order solution $Q_2'(t) = I_2(t)$, where d is now given by the expression

$$d = \frac{e}{m\epsilon} Q_2'(t) \frac{(\Delta t)^3}{6}\left[1 - \frac{3}{4}\frac{Q_2''(t)}{Q_2'(t)}\Delta t\right] \quad (31)$$

and $V(t)$ becomes

$$V(t) = \frac{e}{m\epsilon^2} Q_2'^2(t) \frac{(\Delta t)^4}{8}\left[1 - \frac{6}{5}\frac{Q_2''(t)}{Q_2'(t)}\Delta t\right]. \quad (32)$$

Combining (31) and (32) and expanding into a series, which will be convergent as the second term in the bracket is small in comparison with 1, we obtain, after dropping higher-order terms, the following equations:

$$Q_2'(t) = Q_1'(t)\left[1 + \frac{3}{10}\frac{Q_2''(t)}{Q_2'(t)}\Delta t\right]. \quad (33)$$

Differentiating (31) and (32), and neglecting higher-order terms, we have

$$\frac{d(\Delta t)}{dt} = -\frac{Q_2''(t)}{3Q_2'(t)} \quad (34)$$

$$\frac{dV(t)}{dt} = \frac{1}{12}\frac{e}{m\epsilon^2} Q_2'(t)Q_2''(t)(\Delta t)^4$$

$$= \frac{1}{2\epsilon} d Q_2''(t)\Delta t. \quad (35)$$

Substituting (35) into (33), we have

$$Q_2'(t) = Q_1'(t) + \frac{3}{5}\frac{\epsilon}{d}\frac{dV(t)}{dt}. \quad (36)$$

To speak about the deviation of the static-characteristic-plotting methods, we always mean the deviation of the component of the induced current which results from the motion of the electrons only, because the capacitive component is usually separated out and considered to be in the external circuit. By definition, $Q_c'(t) = (\epsilon/d)dV/dt$. When this is subtracted from the total current $Q_2'(t)$, we call what remains the second-order solution of the induced current resulting from the motion of electrons. Denoting it by $Q_{2i}'(t)$ or $I_{2i}(t)$ where,

$$Q_2'(t) = Q_{2i}'(t) + Q_c'(t), \quad \text{we have}$$

$$Q_{2i}'(t) = Q_1'(t) - \frac{2}{5}\frac{\epsilon}{d}\frac{dV}{dt}. \quad (37)$$

Now in order that the static-characteristic-plotting methods be true, the second-order correction should be small in comparison with the first-order term. Hence we obtain the criterion

$$\left|\frac{2\epsilon}{5d}\frac{dV}{dt}\right| \ll |Q_1'(t)|$$

or

$$\left|\frac{2}{5}c\frac{dV}{dt}\right| \ll |I_1(t)|. \quad (38)$$

From the above analysis, it is clear that the first deviation from the static-characteristic-plotting method is obtained by subtracting 2/5 of the capacitive current. Since the inequality only holds true for large values of I_1, this correction is evidently not sufficient for the region around the cutoff part of the pulse. The result obtained here agrees with that derived in the small-signal theory. The above derivation shows that the same correction can be applied for any kind of signal, large or small, harmonic or nonharmonic. In case of low-frequency small-signal operation, the tube is always conducting, hence this 2/5 negative-capacitance current always exists, and the equivalent circuit can be drawn such that $-2/5$ of the real capacitance forms one branch of the circuit. In case of our large-signal theory, this correction only exists during the period of conduction. Hence, if we consider the fundamental component only, the equivalent negative capacitance should be smaller than $-2/5$ of the real capacitance.

Case (b) Moderately High Frequency $t_1 < t_2$

We shall describe different periods according to the successive diagrams of Fig. 3 (a) and (b).

1. $t < 0$. No electrons are present in the space and hence the induced current resulting from the motion of electrons is zero. Evidently by (1) and the definition of Q, we have

$$Q(t) = Q_c(t) = \epsilon\frac{V(t)}{d}$$

$$Q'(t) = Q_c'(t) = \epsilon\frac{V'(t)}{d}. \quad (39)$$

2. $0 < t < t_1$. Inside the empty space, Laplace's equation holds true, hence the electric field is constant and equal to that for the $\tau = 0$ electron. The potential and the electric field of the $\tau = 0$ electron are $\phi(t, 0)$ and $E(t, 0)$, respectively. The potential drop in the empty space is

$$V(t) - \phi(t, 0) = -[d - x(t, 0)]E(t, 0). \quad (40)$$

By definition, $Q(0) = 0$, and from (12),

$$\epsilon E(t, 0) = -Q(t) + Q(0) = -Q(t). \quad (41)$$

Now in this period, $\tau_{x0} = t$, and we can substitute $\tau = 0$ in (23), giving

$$\phi(t, 0) = \frac{1}{\epsilon}Q(t)x(t, 0) - \frac{e}{m\epsilon^2}\int_0^t \frac{(t-\eta)^2}{2} Q(\eta)Q'(\eta)d\eta. \quad (42)$$

Substituting (41) and (42) into (40), we have

$$V(t) = \frac{1}{\epsilon}Q(t)d - \frac{e}{m\epsilon^2}\int_0^t \frac{(t-\eta)^2}{2} Q(\eta)Q'(\eta)d\eta. \quad (43)$$

Differentiating twice, we obtain the following two equations:

$$V'(t) = \frac{1}{\epsilon} Q'(t)d - \frac{e}{m\epsilon^2}\int_0^t (t-\eta)Q(\eta)Q'(\eta)d\eta \quad (44)$$

$$Q''(t) + aQ^2(t) = \epsilon \frac{V''(t)}{d}. \quad (45)$$

By the definition of $Q_c(t)$ and $Q_c'(t)$, we can separate out the capacitance component from (43) and (44) such that,

$$Q_i(t) = \frac{e}{md\epsilon}\int_0^t \frac{(t-\eta)^2}{2} Q(\eta)Q'(\eta)d\eta \quad (46)$$

$$Q_i'(t) = \frac{e}{md\epsilon}\int_0^t (t-\eta)Q(\eta)Q'(\eta)d\eta. \quad (47)$$

The boundary time t_1 between this period and the next period is characterized by (17) with $\tau_d=0$ and $t=t_1$ or,

$$1 = \frac{e}{md\epsilon}\int_0^{t_1} \frac{(t_1-\eta)^2}{2} Q'(\eta)d\eta. \quad (48)$$

3. $t_1<t<t_2$. Now the whole space is entirely filled by electrons. We can say by (10) and (23), with $x(t, \tau_d)=d$,

$$V(t) = \phi(t, \tau_d) = \frac{1}{\epsilon}Q(t)d - \frac{e}{m\epsilon^2}\int_{\tau_d}^t \frac{(t-\eta)^2}{2} Q(\eta)Q'(\eta)d\eta. \quad (49)$$

We can differentiate this with respect to t, and remembering (20), obtain the expression

$$V'(t) = \frac{1}{\epsilon} Q'(t)d - \frac{e}{m\epsilon^2}\int_{\tau_d}^t (t-\eta)Q(\eta)Q'(\eta)d\eta + \frac{1}{\epsilon} u_d \cdot Q(\tau_d). \quad (50)$$

Separating out the capacitive component again, we have

$$Q_i(t) = \frac{e}{md\epsilon}\int_{\tau_d}^t \frac{(t-\eta)^2}{2} Q(\eta)Q'(\eta)d\eta \quad (51)$$

$$Q_i'(t) = \frac{e}{md\epsilon}\int_{\tau_d}^t (t-\eta)Q(\eta)Q'(\eta)d\eta - \frac{1}{d}u_d Q(\tau_d). \quad (52)$$

Since the electric field is kept zero at the cathode surface during the emission period, the displacement current is zero at the cathode, and the total current is equal to the emission current by (3). At the time $t=t_2$, the emission current becomes zero, or

$$Q(t_2) = Q_c(t_2) + Q_i(t_2) = 0. \quad (53)$$

4. $t_2<t<t_3$. In this period, τ_{x0} is not equal to t but must be determined from (18). Proceeding in a manner similar to the derivation in the previous period, we finally arrive at the following two equations:

$$Q_i(t) = \frac{e}{md\epsilon}\int_{\tau_d}^{\tau_{x0}} \frac{(t-\eta)^2}{2} Q(\eta)Q'(\eta)d\eta \quad (54)$$

$$Q_i'(t) = \frac{e}{md\epsilon}\int_{\tau_d}^{\tau_{x0}} (t-\eta)Q(\eta)Q'(\eta)d\eta - \frac{1}{d}u_d Q(\tau_d) + \frac{1}{d}u_{x0} Q(\tau_{x0}). \quad (55)$$

At the boundary time $t=t_3$ the last electron τ_{dl}, barely touches the plate and turns back. Hence the velocity of this τ_{dl} electron is zero at this particular instant, or by (19), we have

$$0 = \frac{e}{m\epsilon}\int_{\tau_{dl}}^{t_3} (t_3-\eta)Q'(\eta)d\eta. \quad (56)$$

By definition of τ_{dl} and (17), we have

$$1 = \frac{e}{md\epsilon}\int_{\tau_{dl}}^{t_3} \frac{(t_3-\eta)^2}{2} Q'(\eta)d\eta. \quad (57)$$

Equations (56) and (57) must be solved simultaneously to obtain the values of the time t_3 and the constant τ_{dl}.

5. $t_3<t<t_4$. The tube is again only partially filled by electrons. Similar to the second period, we can divide the potential drop in the space into two parts. The boundary between them is characterized by the τ_{dl} electron. The final results of the mathematical manipulation are as follows:

$$Q_i(t) = \frac{e}{md\epsilon}\int_{\tau_{dl}}^{\tau_{x0}} \frac{(t-\eta)^2}{2} Q(\eta)Q'(\eta)d\eta - \frac{1}{d}\cdot x(t, \tau_{dl})\cdot Q(\tau_{dl}) \quad (58)$$

$$Q_i'(t) = \frac{e}{md\epsilon}\int_{\tau_{dl}}^{\tau_{x0}} (t-\eta)Q(\eta)Q'(\eta)d\eta - \frac{1}{d}u(t, \tau_{dl})Q(\tau_{dl}) + \frac{1}{d}u_{x0}Q(\tau_{x0}). \quad (59)$$

The boundary time t_4 is characterized by (18) when the last electron reaches the cathode,

$$0 = \frac{e}{m}\int_{\tau_{dl}}^{t_4} \frac{(t_4-\eta)^2}{2} Q'(\eta)d\eta. \quad (60)$$

6. $t>t_4$. The space is again empty and like the first period, we have

$$Q(t) = Q_c(t) = \epsilon \frac{V(t)}{d}$$

$$Q'(t) = Q_c'(t) = \epsilon \frac{V'(t)}{d}. \quad (61)$$

Case (c) High Frequency $t_1>t_2$

This case differs from the above case by the period 3 if we compare Fig. 3 (b) with Fig. 3 (c). We shall list the equations as follows:

1. $t<0$. Same as case (b) 1.

2. $0<t<t_2$. Same as case (b) 2, except that the boundary between this period and the next is given by

$$Q(t_2) = 0. \tag{62}$$

3. $t_2<t<t_1$. The two equations in this period are

$$Q_i(t) = \frac{e}{mde} \int_0^{\tau_{x0}} \frac{(t-\eta)^2}{2} Q(\eta)Q'(\eta)d\eta \tag{63}$$

$$Q_i'(t) = \frac{e}{mde} \int_0^{\tau_{x0}} (t-\eta)Q(\eta)Q'(\eta)d\eta + \frac{1}{d} u_{x0} Q(\tau_{x0}). \tag{64}$$

The boundary time t_1 is again characterized by the formula,

$$1 = \frac{e}{mde} \int_0^{t_1} \frac{(t_1-\eta)^2}{2} Q'(\eta)d\eta. \tag{65}$$

4. $t_1<t<t_3$.
5. $t_3<t<t_4$.
6. $t_4<t$.

} Same as cases (b) 4, 5, and 6, respectively.

Case (d) Very High Frequency

This case is represented in Fig. 3 (d). We have a situation very similar to case (c) except with several periods omitted.

1. $t<0$. Same as case (c) 1.

2. $0<t<t_2$. Same as case (c) 2.

3. $t_2<t<t_4$. Same as case (c) 3. t_4 is given by the equation

$$0 = \frac{e}{mde} \int_0^{t_4} \frac{(t-\eta)^2}{2} Q'(\eta)d\eta. \tag{66}$$

4. $t_4<t$. Same as case (c) 6.

It is noted that all the above derived formulas involving Q_i' can be checked directly by the image method through the use of (1).

By examining the equations of different periods of the high-frequency cases when the tube is conducting, we see different nonlinear integral equations. In period 2 only, in all the three cases, have we only one unknown function Q for which to solve. For the other periods, we have to solve simultaneously for two or three unknown functions such as Q, τ_d, and τ_{x0} from several nonlinear integral equations. Therefore only in the period 2, can we have a series development of the nonlinear type while in the other periods we must use some numerical integration methods to carry out the solution. We can obtain Taylor's series for the function Q expanding around the origin $t=0$, if we know the successive derivatives of the function Q at the origin. Let us call them $Q_0^{(n)} = (d^nQ(t)/dt^n)_{t=0}$. From the differential equation (45), we obtain the relation between the function Q and its first and second derivatives. By successive differentiation of (45), we can obtain the following recursion formulas which interlink all the higher derivatives with the lower ones:

For n = odd,

$$Q_0^{(n+2)} = \epsilon \frac{V^{(n+2)}(0)}{d} - 2a \left[Q_0Q_0^{(n)} + nQ_0^{(1)}Q_0^{(n-1)} + \cdots \frac{n(n-1)\cdots\frac{n+3}{2}}{\frac{n-1}{2}!} Q_0^{\left(\frac{n-1}{2}\right)}Q_0^{\left(\frac{n+1}{2}\right)} \right].$$

For n = even,

$$Q_0^{(n+2)} = \epsilon \frac{V^{(n+2)}(0)}{d} - 2a \left[Q_0Q_0^{(n)} + nQ_0^{(1)}Q_0^{(n-1)} + \cdots \frac{n(n-1)\cdots\left(\frac{n}{2}+1\right)}{2\left(\frac{n}{2}\right)!} Q_0^{(n/2)}Q_0^{(n/2)} \right]. \tag{67}$$

Starting from the two lowest ones which are known quantities as,

$$Q_0 = 0 \text{ by definition, and}$$

$$Q_0^{(1)} = \epsilon \frac{V'(0)}{d} \text{ by (39),}$$

we can evaluate all the higher derivatives by repeated use of the formulas of (67). To discuss the numerical integration we shall take case (b) as an example. After we have obtained the series in the initial building-up period, we know the functions Q and Q' from time $t=0$ to $t=t_1$. If we want to calculate the function Q at the instant $t=t_1+\Delta t$, where Δt is a small increment of time, we proceed as follows. The $((t-\eta)^2/2)\,Q'(\eta)$ function is plotted from $\eta=0$ to $\eta=t_1$. Between $\eta=t_1$ and $\eta=t_1+\Delta t=t$, the function is unknown, but at $\eta=t_1+\Delta t=t$ the integrand vanishes. If the increment Δt is small, we can make a continuation of the curve from the known section to the unknown section by forcing the curve through the two end points. Then by (17) we can obtain the integration limit τ_d by one of the commonly used numerical integration methods such as Simpson's rule and planimeter. When τ_d is known, we can evaluate $Q_i(t)$ and $Q_i'(t)$ by (54) and (55) in a similar way because the unknown sections of the integrands can be considered as known because they all possess the property of being vanishing at the end point $\eta=t$. After the function at $t=t_1+\Delta t$ has been determined, we can consider the function to be known from $t=0$ to $t=t_1+\Delta t$. By repeating the whole procedure again, we can calculate the function at a later time $t_1+2\Delta t$ as well. This step-by-step integration can be carried out to find all the

values of the Q and Q' functions during a complete cycle of operation by using proper equations for different periods as listed before. When a systematic tabulation is made, this numerical integration is not tedious. By four sets of integration curves (only three are necessary but we use four to obtain Q and Q' separately for a mutual check), we can tell at every instant the position, velocity, and acceleration of every electron emitted from the cathode as well as the induced current due to the motion of electrons, plate dissipation, filament bombardment, and emission current.

Special Examples

In an ordinary class C amplifier, we usually have sinusoidal voltages biased to different degrees. It involves too much work to give all the solutions for different values of bias voltage. Here we give two extreme cases, the first corresponding to the highly biased case) and the second corresponding to the zero-biased class B amplifier. For the highly biased case, we can assume a parabolic voltage for approximation. A complete analysis is given for the parabolic case. We can draw certain conclusions from the results of such a special case because for all other biased voltages, the general nature of the solutions are still the same although they will differ in details. The series expansions of the building-up period for the two cases are given below.

(a) Parabolic voltage $V(t) = V_m(2T - T^2)$

$$Q_c(t) = \epsilon \frac{2V_m}{d}(T - \tfrac{1}{2}T^2) \tag{68}$$

$$\begin{aligned} Q_i(t) = \epsilon \frac{2V_m}{d}\Bigg\{ & C\left(\frac{T^4}{4!} - \frac{3T^5}{5!} + \frac{3T^6}{6!}\right) \\ & + C^2\left(\frac{5T^7}{7!} - \frac{33T^8}{8!} + \frac{84T^9}{9!} - \frac{84T^{10}}{10!}\right) \\ & + C^3\left(\frac{75T^{10}}{10!} - \frac{855T^{11}}{11!} + \frac{4089T^{12}}{12!} - \frac{9702T^{13}}{13!} + \frac{9702T^{14}}{14!}\right) \\ & + C^4\left(\frac{2475T^{13}}{13!} - \frac{31545T^{14}}{14!} + \frac{333060T^{15}}{15!}\right. \\ & \qquad \left. - \frac{1293812T^{16}}{16!} + \frac{3182256T^{17}}{17!} - \frac{3182256T^{18}}{18!}\right) \\ & + \cdots \Bigg\}. \end{aligned} \tag{69}$$

(b) Half sine voltage $V(t) = V_m \sin\theta$

$$Q_c(t) = \epsilon \frac{V_m}{d} \sin\theta \tag{70}$$

$$\begin{aligned} Q_i(t) = \epsilon \frac{V_m}{d}\Bigg\{ & K\left(\frac{2\theta^4}{4!} - \frac{8\theta^6}{6!} + \frac{32\theta^8}{8!}\right. \\ & \qquad \left. - \frac{128\theta^{10}}{10!} + \frac{512\theta^{12}}{12!} - \cdots\right) \\ & + K^2\left(\frac{20\theta^7}{7!} - \frac{252\theta^9}{9!} + \frac{2424\theta^{11}}{11!} - \frac{22088\theta^{13}}{13!} + \cdots\right) \\ & + K^3\left(\frac{600\theta^{10}}{10!} - \frac{16560\theta^{12}}{12!} + \cdots\right) \\ & + K^4\left(\frac{39600\theta^{13}}{13!} - \cdots\right) \\ & + \cdots \Bigg\}. \end{aligned} \tag{71}$$

From dimensional analysis, it is evident that for a fixed wave form of the voltage or for a fixed bias of a sinusoidal voltage, the mode of operation depends upon one normalization constant, such as C and K in the above cases. The numerical relation between the constant C and the wavelength λ or the frequency f is given by the following formulas:

$$\left.\begin{aligned} f &= 0.593 \cdot 10^6 \sqrt{\frac{V_m}{C}}\,\frac{\gamma}{d} \text{ cycles per second} \\ \lambda &= 506 \sqrt{\frac{C}{V_m}}\,\frac{d}{\gamma} \text{ meters} \end{aligned}\right\} \tag{72}$$

where V_m is in volts and d is in meters.

The results for the parabolic voltage are plotted in Figs. 4, 5, and 6. All curves are plotted with the corresponding maximum value at low frequency, i.e., $C = \infty$, as a unit. According to Fig. 2 the five different curves with $C = 52.1$, 13.9, 7.57, 4.69, and 3.18 correspond to case (b) operation. The curves with $C = 1$ correspond to case (c) operation and the curves with $C = 0.6$ to case (d) operation. From Fig. 4, we see that the induced current can be negative during the part of the cycle when the backward-moving electrons exercise greater influence than the forward-moving ones. During a part of a cycle the electrons are actually delivering energy to the source. This is possible through the storage of kinetic energy in the space charge in the previous period. Therefore, the equivalent circuit of the diode at large signals will also involve inductive reactance. As the frequency increases, the negative peak current increases at first, up to a certain frequency, and then decreases again when the mode of operation is case (d) because electrons have less chance to fill in the whole tube space as a result of the rapid reversal of voltage. In Fig. 5 we see that the conduction current at the plate gradually lags behind the voltage and the interval during which the plate receives electrons decreases as frequency increases. Up to a frequency corresponding to $C = 0.7$, i.e., the boundary between case (c) and case (d) operation, no electrons will reach the plate during the complete cycle of operation. This boundary value forms the ultimate limit of the pulse-production mechanism. Fig. 6 shows the velocity of the electrons at the plate and the velocity of the returning electrons at the cathode.

III. Intergrids or Grid-Plate Space of a Multiple-Electrode Tube

In this section, we shall deal with a plane diode in which there is a periodical injection of electronic current at the first electrode of specified charge density $\rho_0(t)$ and velocity $u_0(t)$. For example, in the analysis of the plate-grid space of a triode, these are given by the distribution curves of Figs. 5 and 6 for the equivalent diode of the cathode-grid space. The basic electronics equations (2) to (7) are the same as in the previous section but the boundary conditions are different. Let us adopt the same symbols and notations as in section II except that we have no cathode now but instead we have the first electrode at which the electronic current is injected. The distances and potentials are measured from this first electrode. The time origin is put at the instant when the first electron is injected. The voltage difference between the two electrodes $V(t)$ is not necessarily zero in this case at $t=0$. Let us define two additional quantities as follows:

$$E_{00} = E(0, 0) = E[0, 0] = -\frac{V(0)}{d} \tag{73}$$

$$\omega(t) = \int_0^t \rho_0(\eta) u_0(\eta) d\eta + \epsilon E_{00}. \tag{74}$$

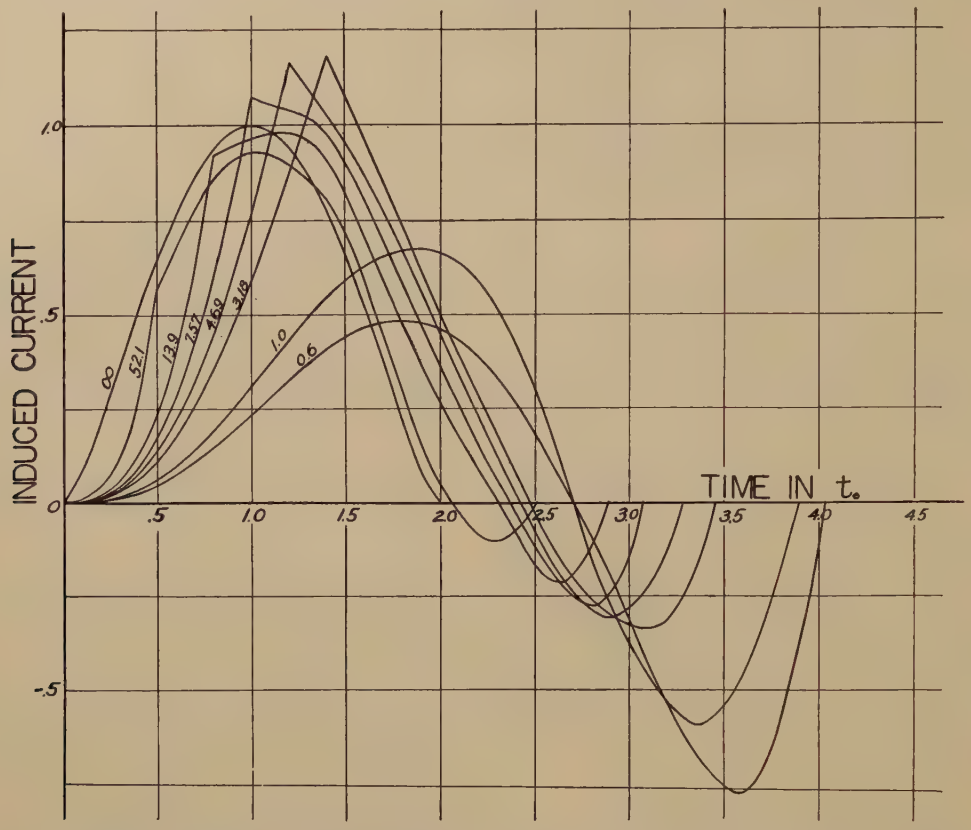


Fig. 4—Current induced in a plane diode from the motion of the electrons versus time, when a parabolic voltage is applied. Parameter $= C$. Unit of current = the maximum current at $C = \infty$.

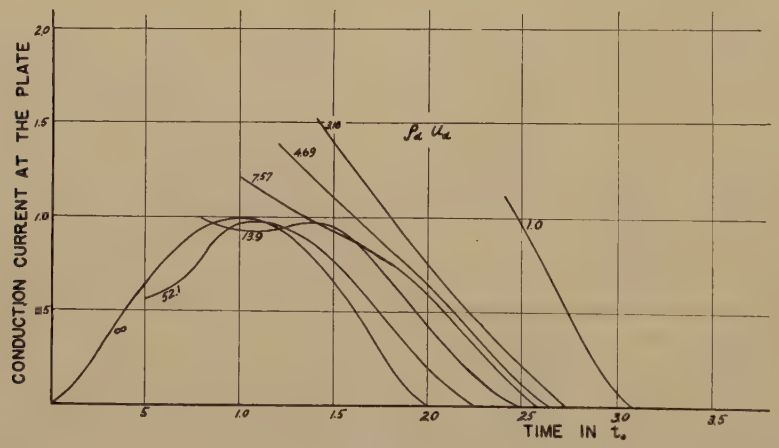


Fig. 5—Conduction current at the plate $\rho_d u_d$ versus time when a parabolic voltage is applied to a plane diode. Parameter $= C$. Unit of current = the maximum current at $C = \infty$.

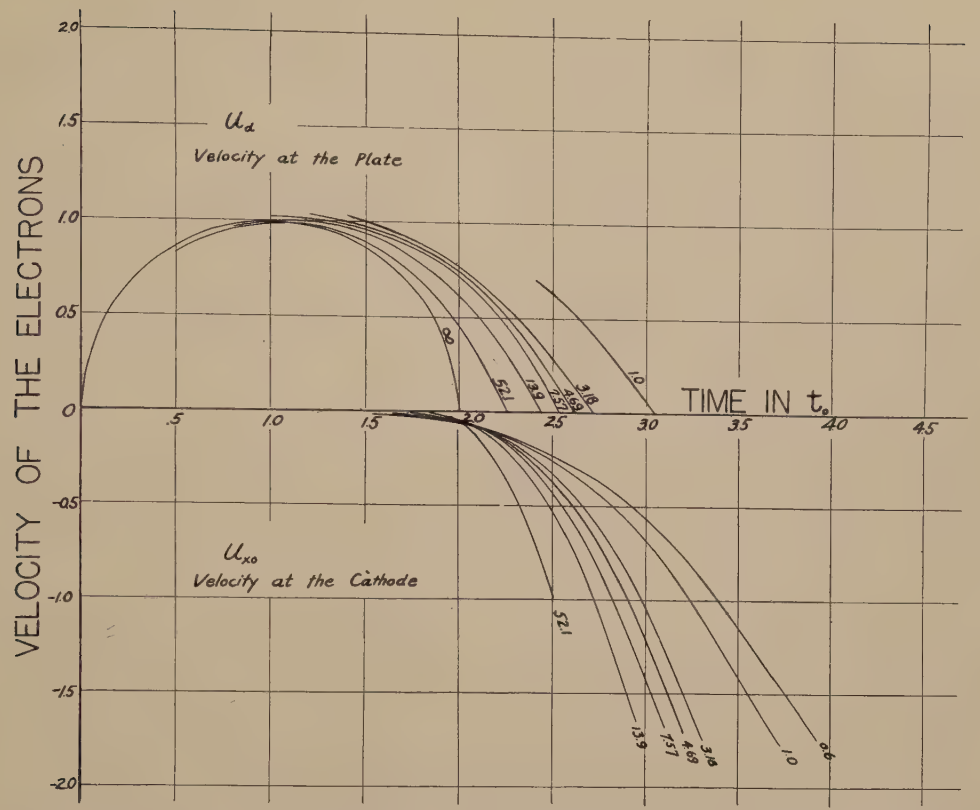


Fig. 6—The velocity of electrons at the plate u_d and the velocity of electrons at the cathode u_{x0} when a parabolic voltage is applied to a plane diode. Parameter $= C$. Unit of velocity = the maximum velocity of electrons at the plate at $C = \infty$.

The boundary conditions are

$$\phi[t, d] = V(t) \tag{75}$$

$$\epsilon \left.\frac{\partial E[t, x]}{\partial x}\right|_{x=0} = -\rho_0(t) \tag{76}$$

$$\left.\frac{\partial x(t, \tau)}{\partial t}\right|_{x=0} = u_0(t) \tag{77}$$

$$x(\tau, \tau) = 0 \tag{78}$$

Let us put $x = 0$ in (3)

$$-\rho_0(t)u_0(t) + \epsilon \frac{\partial E[t, 0]}{\partial t} = -I(t) = -Q'(t). \tag{79}$$

Integrating from $t = 0$ to $t = t$ and remembering (74) and the definition of Q, we have

$$\epsilon E[t, 0] = \epsilon E(t, t) = -Q(t) + \omega(t) \tag{80}$$

or

$$\epsilon E(\tau, \tau) = -Q(\tau) + \omega(\tau). \tag{81}$$

Integrating (6) with respect to t from $t = \tau$ to $t = t$, we have

$$\epsilon[E(t, \tau) - E(\tau, \tau)] = -Q(t) + Q(\tau). \tag{82}$$

Combination of (81) and (82) gives,

$$\epsilon E(t, \tau) = -Q(t) + \omega(\tau). \tag{83}$$

Substituting this into (5) and performing a series of operations similar to those in section II but with a different set of boundary conditions for (75) to (78), we obtain the following formulas:

$$u(t, \tau) = \frac{\partial x(t, \tau)}{\partial t} = u_0(\tau) - \frac{e}{m\epsilon}(t - \tau)\omega(\tau) + \frac{e}{m\epsilon}\int_\tau^t Q(\eta)d\eta \tag{84}$$

$$x(t, \tau) = u_0(\tau)(t - \tau) - \frac{1}{2}\frac{e}{m\epsilon}(t - \tau)^2\omega(\tau) + \frac{e}{m\epsilon}\int_\tau^t (t - \eta)Q(\eta)d\eta \tag{85}$$

$$\frac{\partial x(t, \tau)}{\partial \tau} = \frac{\partial u_0(\tau)(t - \tau)}{\partial \tau} - \frac{\partial}{\partial \tau}\left[\frac{1}{2}\frac{e}{m\epsilon}(t - \tau)^2\omega(\tau)\right] - \frac{e}{m\epsilon}(t - \tau)Q(\tau) \tag{86}$$

$$\phi(t, \tau) = \frac{1}{\epsilon}Q(t)x(t, \tau) - \int_t^\tau \frac{1}{\epsilon}\omega(\eta)\frac{\partial x(t, \eta)}{\partial \eta}d\eta \tag{87}$$

$$d = u(\tau_d)(t - \tau_d) - \frac{e}{2m\epsilon}(t - \tau_d)^2\omega(\tau_d) + \frac{e}{m\epsilon}\int_{\tau_d}^t (t - \eta)Q(\eta)d\eta. \tag{88}$$

Solution at High Frequencies

It is the general concept in the low-frequency analysis of vacuum tubes, that the plate current is equal

to the current injected through the grid unless the current is partially or completely reflected by the formation of a virtual cathode. In high-frequency operation, this is not true. Let us limit our attention to the cases only in which all the electrons injected at the first electrode will arrive at the second electrode. This will be true when the voltage of the second electrode is sufficiently high, the distance between the two electrodes is sufficiently small, and the velocity of the injected electrons is large enough, so that there will be no virtual cathode formed in the space. When this is true, we can divide the complete cycle of operation into four periods. Let t_1 be the time at which the first electron injected reaches the plate and t_3 be the time at which the last electron reaches the plate. The period before t_1 and the period after t_3 have no electrons in the space and hence the current is purely capacitive. The other two periods will be described as follows:

(1) $0<t<t_1$. The building-up period.

The tube is partially filled with electrons. Similar to the method used in section II for case (b) 2, we derive from the above equations the integral equation for the building-up period, as

$$\epsilon F(t) = Q(t) - \frac{e}{md\epsilon}\int_0^t (t-\eta)[\omega(\eta) - \epsilon E_{00}]Q(\eta)d\eta. \quad (89)$$

Where $F(t)$ is a known function given by the equation,

$$F(t) = \frac{1}{d}\left\{V(t) + E_{00}d - \frac{1}{\epsilon}\int_0^t [\omega(\eta) - \epsilon E_{00}]\frac{\partial}{\partial\eta}\left[(t-\eta)u_0(\eta) - \frac{e}{2m\epsilon}(t-\eta)^2\omega(\eta)\right]d\eta\right\}. \quad (90)$$

By differentiating (89) and separating out the capacitive current we have

$$Q_i'(t) = \frac{1}{d}\int_0^t \omega'(\eta)u(t,\eta)d\eta \quad (91)$$

which can also be obtained directly by the image method. The boundary time t_1 is obtained by the formula

$$1 = u_0(0)t_1 - \frac{e}{2md\epsilon}\omega(0)t_1^2 + \frac{e}{md\epsilon}\int_0^{t_1}(t_1-\eta)Q(\eta)d\eta. \quad (92)$$

We see that (90) is a linear integral equation of Volterra type. The solution can be found in different mathematical textbooks for integral equations. One method is to expand the solution into Neumann's series.[9] The Neumann development is theoretically possible for any regular functions of $\omega(\eta)$ and $F(t)$, but it is too laborious for practical use. To simplify the solution, this integral equation can be converted into a differential equation which, for a certain restriction on the value of $\omega(\eta)$, can be solved by an explicit series development. This differential equation is

$$Q_1''(t) - \frac{e}{md\epsilon}\omega_1(t)Q_1(t) = \epsilon\frac{V''(t)}{d} + \frac{u_0(t)\omega_1'(t)}{d} - \frac{e}{2md\epsilon}\omega_1^2(t) \quad (93)$$

where

$$Q_1(t) = Q(t) - \epsilon E_{c0}$$
$$\omega_1(t) = \omega(t) - \epsilon E_{00}. \quad (94)$$

The initial conditions for this differential equation are

$$Q_1(0) = -\epsilon E_{00} = \epsilon\frac{V(0)}{d}$$
$$Q_1'(0) = \epsilon\frac{V'(0)}{d}. \quad (95)$$

Equation (93) is a linear differential equation of the form

$$\frac{d^2y}{dx^2} + P(x)y = R(x). \quad (96)$$

If $P(x) = -px^k$

$$R(x) = r_0 + r_1x + r_2x^2 + r_3x^3 + r_4x^4 + \cdots$$

and $y(0) = A$, $y'(0) = B$

then the solution of (96) can be obtained by the successive-approximation method as follows:

$$\begin{aligned} y_n = A + Bx &+ r_0\sum_{\nu=1}^{\nu=n} p^{\nu-1}x^{(k+2)(\nu-1)+2} \prod_{\sigma=1}^{\sigma=\nu}\frac{1}{[(k+2)(\sigma-1)+1][(k+2)(\sigma-1)+2]} \\ &+ r_1\sum_{\nu=1}^{\nu=n} p^{\nu-1}x^{(k+2)(\nu-1)+3} \prod_{\sigma=1}^{\sigma=\nu}\frac{1}{[(k+2)(\sigma-1)+2][(k+2)(\sigma-1)+3]} \\ &+ r_2\sum_{\nu=1}^{n} p^{\nu-1}x^{(k+2)(\nu-1)+4} \prod_{\sigma=1}^{\sigma=\nu}\frac{1}{[(k+2)(\sigma-1)+3][(k+2)(\sigma-1)+4]} \\ &+ \cdots \\ &+ A\sum_{\nu=1}^{\nu=n} p^{\nu}x^{(k+2)\nu} \prod_{\sigma=1}^{\sigma=\nu}\frac{1}{[(k+2)\sigma-1][(k+2)\sigma]} \\ &+ B\sum_{\nu=1}^{\nu=n} p^{\nu}x^{(k+2)\nu+1} \prod_{\sigma=1}^{\sigma=\nu}\frac{1}{[(k+2)\sigma][(k+2)\sigma+1]}. \end{aligned} \quad (97)$$

[9] B. Riemann und H. Weber, "Differential Gleichungen der Physik," Friedr. Vieweg und Sohn Akt.-Ges., Braunschweig, 1925, vol. 1, p. 426.

Then $y=\mathrm{Lim}_{n\to\infty}\ y_n$, if such a limit exists.

(2) $t_1<t<t_3$. This period can really be considered to consist of two periods because after the last electron is injected, one part of the tube space is empty. But mathematically it makes no difference since we can consider the function $\rho_0(t)$ to be zero after the last electron is injected. The equations for this period are

$$V(t)-\frac{1}{\epsilon}\int_{\tau_d}^{t}\omega(\eta)\frac{\partial}{\partial n}\left\{(t-\eta)u_0(\eta)-\frac{e}{2m\epsilon}(t-\eta)^2\omega(\eta)\right\}d\eta$$

$$=\frac{1}{\epsilon}Q(t)d-\int_{\tau_d}^{t}\frac{e}{m\epsilon^2}(t-\eta)\omega(\eta)Q(\eta)d\eta \qquad (98)$$

$$Q_i'(t)=\frac{1}{d}\int_{\tau_d}^{t}\omega'(\eta)u(t,\eta)d\eta. \qquad (99)$$

Equation (99) can be checked directly by the image method using (1).

Equation (98) can be solved by numerical integration by a method similar to that used in section II. We would have to solve for two unknown functions Q and τ_d simultaneously, but this involves too much labor. Besides, no general solution can be worked out as we did in the previous section because the injection current can have any arbitrary wave form. For practical purposes, (91) and (99) can be used to estimate the current induced by the moving electrons. This approximate method is sufficiently accurate for most cases of power tubes when the space-charge effect in the plate-grid space is not very great making it possible to form a very good estimate of the velocity of electrons $u(t,\tau)$. Equations (91) and (99) have very important physical significance because they show that the induced current is some sort of weighted average of the injected current over the time period of transit. The above analysis has been applied by the author to several power tubes to show the decrease of output and efficiency, the increase of phase angle between the plate and the grid voltage for optimum operation of the tube, change of load impedance, additional grid-circuit loading, filament bombardment, emission current limit, etc., as the operating frequency increases.

The space-charge distribution between two parallel plates of fixed voltage ratio in the stationary-state operation has been worked out by different authors.[10,11] We have developed the complete theory in analyzing the initial building-up phenomena of such space-charge distributions when a constant space charge and constant-velocity electron stream is suddenly injected. This involves twelve functions derived from (97). Four of the functions have been identified with the Bessel functions of the $\pm\frac{1}{3}$ and $\pm\frac{2}{3}$ order with imaginary argument. The results will not be published for lack of space. Incidentally, this solution will degenerate to Grünberg's case[7] when we allow the initial velocity and the space charge of the injected electron stream to approach zero and infinite values, respectively.

IV. Conclusion

From the theories developed in the previous sections, it is evident that the conventional power tubes have the following limitations:

(a) The scheme of grouping electrons into periodically flowing streams is not successful at high frequency. The relative period in which the electron streams pass through the grid wires into the plate-grid space and the relative interval in which the cathode emits electrons decrease as the frequency increases. Ultimately, no electrons reach the plate space in a highly biased amplifier and consequently no power conversion is possible. This ultimate limit can be calculated.

(b) The effect of the transit time in the grid-plate space is to reduce the possibility of inducing periodic pulses in the plate circuit even if the grid-cathode space can supply properly grouped, periodically flowing electron streams into the grid-plate space. Theoretically speaking, it is possible to decrease this effect by using high-velocity electron streams but this requires high grid voltage. When the grid voltage is high, more electrons will be intercepted by the grid instead of going into the grid-plate space to do the conversion work. Therefore, there will be a limit to the possible maximum positive voltage of the grid which sets the limit of pulse production in the grid-plate space.

(c) Excessive loading on the exciter is likely to occur at high frequency even if no electrons reach the grid. If the plate power produced is less than that required to excite the grid, the power tube ceases to act as a power amplifier.

The newly developed velocity-variation tube utilizes a new principle of grouping electrons such that least driving power is required. This is possible because the high-frequency signal is not called upon to supply the kinetic energy of the moving electrons but simply to modify the velocity by a little amount. The main part of the kinetic energy is supplied by another high-voltage direct-current source and it is later recovered as high-frequency energy. This scheme certainly avoids some of the difficulties encountered in the conventional tubes. However, the wave form of the injection current by bunching electrons in a velocity-variation tube is not ideal and it has a theoretical limit of 58 per cent in efficiency.[12] It is the aim of this paper to stimulate the discussion of the possible improvement of the conventional power tube so that by retaining its simplicity of construction and principles of operation it might be possible to deliver power comparable to

[10] C. E. Fay, "Theory of space charge between parallel plane electrodes," *Bell Sys. Tech. Jour.*, vol. 17, pp. 49–79; January, 1938.

[11] B. Salzberg and A. V. Haeff, "Effects of space charge in the grid-anode region of vacuum tubes," *RCA Rev.*, vol. 2, pp. 336–374; January, 1938.

[12] D. L. Webster, "Cathode ray bunching," *Jour. Appl. Phys.*, vol. 10, pp. 501–513; July, 1939.

those of the newly developed tubes with reasonable efficiency.

V. Acknowledgment

The author takes this opportunity to express his sincere appreciation to Professor E. L. Chaffee, under whose able direction this research has been carried out. He is also indebted to Professor R. King for consultations in high-frequency circuit analysis and to Mr. G. Bergson for his kindness in reading the proofs. Finally, he must thank Mr. L. B. Heilprin for lending his equipment.

High-Frequency Radio Transmission Conditions, April 1941, With Predictions for July 1941*

NATIONAL BUREAU OF STANDARDS, WASHINGTON, D. C.

THE radio transmission data herein are based on observations at Washington, D. C., of long-distance reception and of the ionosphere. Fig. 1 gives the April average values of maximum usable frequencies, for undisturbed days, for radio transmission by way of the regular layers of the ionosphere. The maximum usable frequencies were determined by the F layer at night and by the F_2 layer during the day. Fig. 2 gives the expected values of the maximum usable frequencies for radio transmission by way of the regular layers, average for undisturbed days, for July, 1941. Average critical frequencies and virtual heights of the ionospheric layers as observed at Washington, D. C., during April are given in Fig. 3. Critical frequencies for each day of the month are given in Fig. 4.

Ionospheric storms are listed in Table I. The details of the ionospheric-storm day of April 24 are shown in Fig. 3. The open circles in Fig. 4 indicate the noon and midnight critical frequencies observed during the ionospheric storms listed in Table I. The sizes of the circles roughly represent the severity of the storm. April was characterized by much sustained, although not very severe, ionospheric storminess. About 60 per cent of the hours of the month were disturbed.

TABLE I
Ionospheric Storms (approximately in order of severity)

Day and hour E.S.T.	h_F before sunrise (km)	Minimum f^0_F before sunrise (Mc)	Noon $f^0_{F_2}$ (Mc)	Magnetic character[1] 00–12 G.M.T.	Magnetic character[1] 12–24 G.M.T.	Ionospheric character[2]
April						
23 (from 1800)	—	—	—	0.0	0.0	0.5
24	342	diffuse	<4.2	1.1	1.4	1.5
25	390	<1.6	5.5	1.1	0.6	1.0
26 (through 1200)	350	1.5	6.2	0.6	0.1	0.5
26 (from 1800)	—	—	—	0.6	0.1	0.5
27	340	2.4	5.1	0.0	0.0	1.0
28	312	2.6	4.9	0.4	0.8	1.0
29	390	<1.6	5.0	0.8	0.0	1.5
30	343	1.6	5.2	0.0	0.0	1.0
18 (from 1800)	—	—	—	0.4	0.6	0.5
19	385	2.2	4.7	1.0	0.6	1.0
20	[3]	[3]	[3]	0.5	0.4	[3]
21	[3]	[3]	[3]	0.4	0.1	[3]
22 (through 1400)	336	2.9	5.7	0.1	0.1	0.5
1	[4]	<1.6	4.4	0.1	0.1	1.5
2	325	1.8	5.9	0.3	0.3	1.0
3	315	1.9	5.1	0.6	0.4	1.0
4	320	2.0	5.4	0.1	0.0	1.0
5 (through 0500)	328	1.6	—	0.1	0.1	0.5
6 (from 2200)	—	—	—	0.3	0.2	0.5
7 (through 1800)	330	2.0	5.8	0.3	0.8	1.0
9 (from 2200)	—	—	—	0.6	0.5	0.5
10 (through 0500)	338	2.4	—	0.6	0.9	0.5
11 (0100 through 0500)	328	3.0	—	0.9	0.5	0.5
For comparison: Average for undisturbed days	298	2.79	7.17	0.3	0.2	0.0

[1] American magnetic character figure, based on observations of seven observatories.
[2] An estimate of the severity of the ionospheric storm at Washington on an arbitrary scale of 0 to 2, the character 2 representing the most severe disturbance.
[3] Recorder not in operation.
[4] No regular reflections above 2.5 megacycles.

TABLE II
Sudden Ionospheric Disturbances

Day	G.M.T. Beginning	G.M.T. End	Locations of transmitters	Relative intensity at minimum[1]
April				
7	1624	1640	Ohio, Ont., D. C.	0.1
18	2238	2300	Ohio, Ont., D. C.	0.1
19	1851	1910	Ohio, Ont., D. C.	0.02

[1] Ratio of received field intensity during fade-out to average field intensity before and after, for station W8XAL, 6080 kilocycles, 650 kilometers distant.

The sudden ionosphere disturbances are listed in Table II.

TABLE III
Approximate Maximum Usable Frequencies in Megacycles, for Radio Transmission via Strong Sporadic-E Reflections
Hour, E.S.T.

Day	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23
6																		16	18	14				
8																							14	
11																				16	17			
12																			19	17	14			
13																					15	16		
15																					17	23	15	
16																						16		
17												22												
21																				14	21	16	15	18
22	15																							
23																				14				
27															18	17			17	18	19			
29	22	19	20	16	15	14																		

* Decimal classification: R113.61. Original manuscript received by the Institute May 12, 1941. These reports have appeared monthly in the Proceedings starting in vol. 25, September, 1937. See also vol. 25, pp. 823–840: July, 1937. Report prepared by T. R. Gilliland, N. Smith, R. Silberstein, and C. O. Marsh.

Table III gives the approximate maximum usable frequencies for good radio transmission via sporadic-E reflections. It should be noted that this differs from the corresponding table in previous reports (which gave the vertical-incidence frequencies).

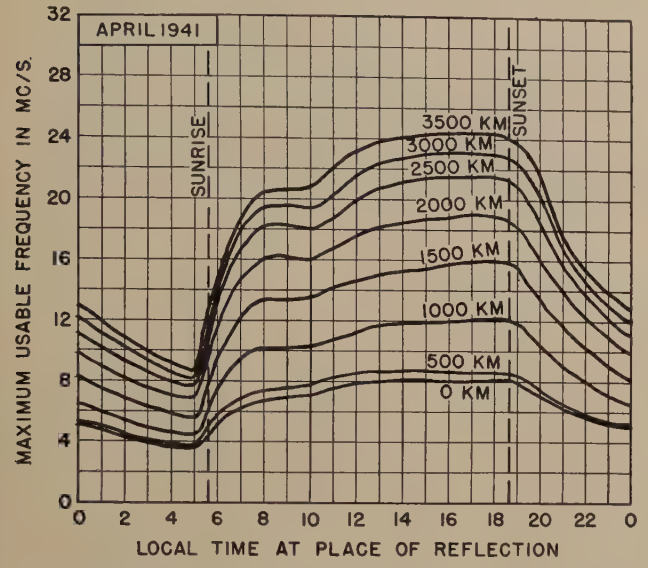


Fig. 1.—Maximum usable frequencies for dependable radio transmission via the regular layers, average for undisturbed days, for April, 1941. These curves and those of Fig. 2 also give skip distances, since the maximum usable frequency for a given distance is the frequency for which that distance is the skip distance.

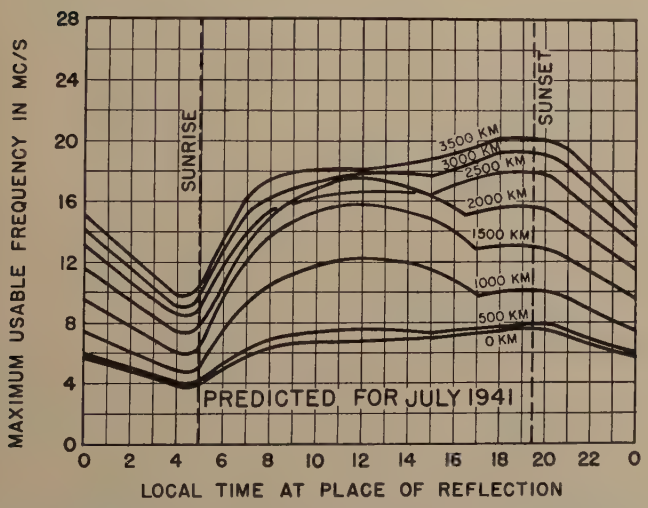


Fig. 2—Predicted maximum usable frequencies for dependable radio transmission via the regular layers, average for undisturbed days, for July, 1941. The values shown will be considerably exceeded during irregular periods by reflections from clouds of sporadic E layer. For information on use in practical radio transmission problems, see Letter Circulars 614 and 615 obtainable from National Bureau of Standards, Washington, D. C., on request.

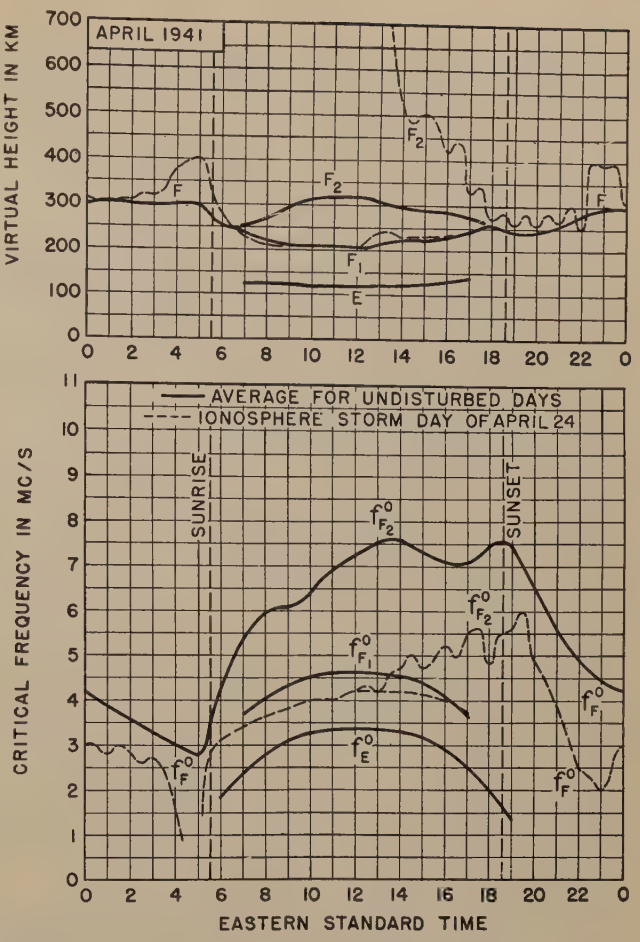


Fig. 3—Virtual heights and critical frequencies of the ionospheric layers, observed at Washington, D. C., April, 1941.

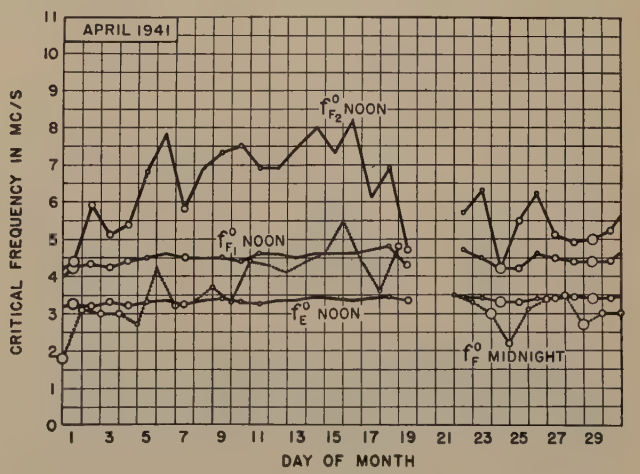


Fig. 4—Midnight $f_F{}^0$ and noon $f_E{}^0$, $f_{F_1}{}^0$, and $f_{F_2}{}^0$, for each day of April. Open circles indicate critical frequencies observed during ionosphere storms. Sizes of circles represent approximately the severity of the storm.

Institute News and Radio Notes

Institute Establishes a Program for Fostering Relations with Universities and University Students

Recognizing that a steadily increasing proportion of the young engineers entering the radio profession have been trained in universities, the Institute has taken steps to strengthen its university and student relations. This action was started when the Board of Directors approved a new Bylaw that provides for the appointment of Institute members who are teachers of communication as INSTITUTE REPRESENTATIVES.

Each such INSTITUTE REPRESENTATIVE is to serve as the representative of the I.R.E. at his school and is charged with promoting the welfare of the Institute particularly in regard to student membership matters. The Institute Representatives are authorized to use the name of the Institute in connection with such activities as student meetings.

Institute Representatives have already been appointed for approximately 65 educational institutions. Additional schools are being added as rapidly as interest is shown in the I.R.E. among the students, and provided there is an I.R.E. member on the teaching staff.

A membership campaign has also been launched to bring into the Institute as many as possible of those teachers of radio and communication who are not already members. These individuals represent very desirable members for the Institute. At the same time the Institute, through its Proceedings, can benefit teachers by enabling them to keep up with current developments, an obviously important factor in effective teaching.

The results that have been obtained to date from the student membership program are extremely encouraging. In 1938, when no special efforts were being made to call the attention of the Institute to college students, there were 314 student members. In 1940 this had increased to 496, as a result of an informal campaign carried on in the Pacific Coast states in 1939, and extended to the entire United States and Canada in 1940. With the new plan of Institute Representatives, a further increase can be expected for 1941.

We have reached the point where the number of new Associate members gained each year by transfer from student status is considerable. These young men who receive training in radio engineering in college, and then go out into radio and communication industries, represent the very finest possible type of new blood for the Institute. They now have much to receive from, and in the future will have much to give to, the Institute.

Frederick Emmons Terman,
President

SUMMER CONVENTION
June 23, 24, and 25, 1941
Detroit, Michigan

The Institute has held its annual conventions for the past several years during the summer. A recent change in policy has resulted in the holding of two conventions each year, with the annual convention being held in New York every winter. The summer convention for 1941 will be held in Detroit, Michigan, and convention headquarters will be at the Hotel Statler.

The program which follows is considered as being final, but it may be modified if conditions require such action. Eastern Standard Time is used in Detroit. Technical sessions will be held on each morning and on Wednesday afternoon. The other two afternoons will be devoted to inspection trips to a number of interesting places in Detroit.

Technical papers are not available in printed form either before or after the convention and there is no assurance that any of these papers will be published in the Proceedings.

PROGRAM

Sunday, June 22

4:00 P.M.–6:00 P.M.

Advance Registration

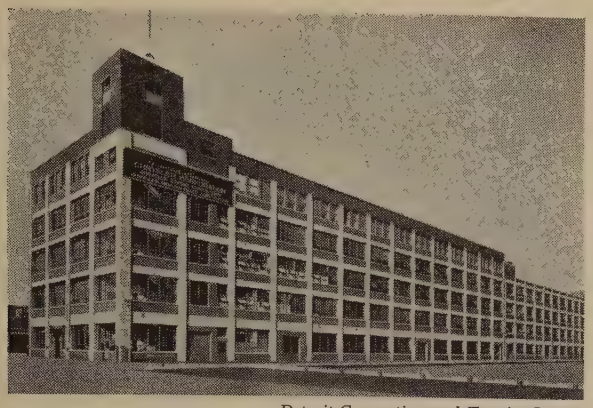
Monday, June 23

8:30 A.M.–5:00 P.M.

Registration

10:00 A.M.–12:30 P.M.

Addresses of welcome by F. E. Terman, President of the Institute; M. Cottrell, Chairman of the Detroit



Detroit Convention and Tourist Bureau

Through the courtesy of the Chrysler Corporation the women guests will be taken for a sightseeing trip terminating at the Chrysler plant where luncheon will be served and motion pictures shown.

Section; and A. B. Buchanan, Convention Committee Chairman.

Technical Session, Banquet Room

1. "Photographic Analysis of Television Images," by D. G. Fink, McGraw-Hill Publishing Company, New York, N. Y. (twenty minutes)
2. "Industrial Electronic Applications," by R. A. Powers, Electronic Control Corporation, Detroit, Mich. (twenty minutes)
3. "A Radio-Frequency Device for Detecting the Passage of a Bullet," by C. I. Bradford, Remington Arms Company, Inc., Bridgeport, Conn. (twenty minutes)
4. "Mobile Television Equipment," by R. L. Campbell, R. E. Kessler, R. L. Rutherford, and K. U. Landsberg, Allen B. Du Mont Laboratories, Inc., Passaic, N. J. (with demonstration, thirty minutes)

10:30 A.M.–11:00 A.M.

Women will meet at their headquarters.

11:00 A.M.–4:45 P.M.

Trip No. 1. Women's sightseeing trip.

2:30 P.M.–5:00 P.M.

Technical Session, Banquet Room

5. "Ultra-High-Frequency Loop Antennas for Frequency-Modulation Broadcasting," by Andrew Alford, A. G. Kandoian, and R. A. Hampshire, International Telephone Development Company, Inc., New York, N. Y. (fifteen minutes)
6. "A Turnstile Antenna for Ultra-High-Frequency Broadcasting," by G. H. Brown and J. Epstein, RCA Manufacturing Company, Inc., Camden, N. J. (fifteen minutes)
7. "Frequency Modulation for Emergency Communication," by F. T. Budelman, F. M. Link Company, New York, N. Y. (fifteen minutes)
8. "A New Frequency-Modulation Transmitter," by N. C. Olmstead and A. A. Skene, Bell Telephone Laboratories, Inc., New York, N. Y. (fifteen minutes)
9. "A Frequency-Modulation Station Monitor," by H. R. Summerhayes, Jr., General Electric Company, Schenectady, N. Y. (fifteen minutes)
10. "Phase Distortion in Frequency-Modulation Systems," by N. I. Korman, RCA Manufacturing Company, Inc., Camden, N. J. (fifteen minutes)

2:30 P.M.–5:00 P.M.

Technical Session, Bagley Room

11. "Control of Night Error in Airplane Direction Finding," by H. Busignies, International Telephone Development Company, Inc., New York, N. Y. (fifteen minutes)
12. "A Method of Changing the Frequency of a Complex Wave," by E. L. Kent, C. G. Conn, Ltd., Elkhart, Ind. (fifteen minutes)
13. "A New Air-Cooled 5-Kilowatt Broadcast Transmitter," by F. W. Fischer, Westinghouse Electric and Manufacturing Company, Baltimore, Md. (fifteen minutes)
14. "Optimum Current Distributions on Vertical Antennas," by Lincoln La Paz and G. A. Miller, Ohio State University, Columbus, Ohio, and National Research Council, Ottawa, Ont., Canada, respectively. (fifteen minutes)
15. "Short-Wave Spread-Band Receiver Circuits," by D. E. Foster and Garrard Mountjoy, Radio Corporation of America, License Division Laboratory, New York, N. Y. (fifteen minutes)
16. "Factory Alignment Equipment for Frequency-Modulation Receivers," by H. E. Rice, Stromberg-Carlson Telephone Manufacturing Company, Rochester, N.Y. (fifteen minutes)
17. "The Full-Wave Voltage-Doubling Rectifier circuit," by D. L. Waidelich, University of Missouri, Columbia, Mo. (fifteen minutes)

7:00 P.M.–9:30 P.M.

Trip No. 2. Harper Hospital. "Electronics in Medicine" will be described and demonstrated by members of the staff. The women are invited to attend this lecture-demonstration.

In the Menlo Park group of buildings at Greenfield Village, the Edison laboratory is of particular interest, for it was here that Edison invented the incandescent light.

Clinton Inn is one of the many interesting places in Greenfield Village.

Tuesday, June 24

9:00 A.M.–1:00 P.M.

Registration

9:30 A.M.–11:30 A.M.

Technical Session, Banquet Room

18. "Transmission of an Electromagnetic Wave by a Row of Equidistant Similar Plates," by Hillel Poritsky, General Electric Company, Schenectady, N. Y. (twenty minutes)
19. "The Field-Theory Approach to Nonuniform Transmission Lines," by Simon Ramo, General Electric Company, Schenectady, N. Y. (twenty minutes)
20. "A Mechanical Calculator for Directional Antenna Patterns," by W. G. Hutton, WGAR, Cleveland, Ohio. (fifteen minutes)
21. "Theory of Radial Direct-Current Space-Charge Flow Between Concentric Cylinders," by W. G. Dow and A. B. Bronwell, University of Michigan, Ann Arbor, Mich., and Northwestern Technological Institute, Evanston, Ill., respectively. (fifteen minutes)
22. "Some Simplified Methods of Determining the Optical Characteristics of Electron Lenses, by Karl Spangenberg and L. M. Field, Stanford University, Stanford University, Calif. (fifteen minutes)

9:30 A.M.–11:30 A.M.

Technical Session, Bagley Room

23. "Deionization Considerations in a Harmonic Producer Employing a Gas-Tube Switch," by W. G. Shepherd, Bell Telephone Laboratories, Inc., New York, N. Y. (fifteen minutes)
24. "Design and Development of Three New Ultra-High-Frequency Transmitting Tubes," by C. E. Haller, RCA Manufacturing Company, Inc., Harrison, N. J. (fifteen minutes)
25. "New Small Ultra-High-Frequency Receiving Tubes," by L. B. Curtis, RCA Manufacturing Company, Inc., Harrison, N. J. (fifteen minutes)
26. "Trends in Receiving-Tube Design," by R. L. Kelly, RCA Manufacturing Company, Inc., Harrison, N. J. (fifteen minutes)
27. "The Effects of Contact Potentials on the Characteristics of Vacuum Tubes," by G. D. O'Neill, Hygrade Sylvania Corporation, Salem, Mass. (fifteen minutes)
28. "A Method of Calculating the Performance of Self-Biased Plate-Modulated Amplifiers," by R. I. Sarbacher, Illinois Institute of Technology, Chicago, Ill. (fifteen minutes)

1:00 P.M.–5:30 P.M.

Trip No. 3. Edison Institute Museum and Greenfield Village.

7:00 P.M.

Banquet and Entertainment, Grand Ballroom.

Wednesday, June 25

9:00 A.M.–12:30 P.M.

Registration

9:30 A.M.–12:00 NOON

Technical Session, Banquet Room

29. "The Relative Sensitivities of Television Pickup Tubes, Photographic Film, and the Human Eye," by Albert Rose, RCA Manufacturing Company, Inc., Harrison, N. J. (twenty minutes)
30. "Measurement, Analysis, Synthesis, and Evaluation of the Square-Wave Response of Television Apparatus," by R. D. Kell, A. V. Bedford, G. L. Frendendall, and H. N. Kozanowski, RCA Manufacturing Company, Inc., Camden, N. J. (with demonstration, thirty minutes)
31. "Observations of Frequency-Modulation Propagation on 26 Megacycles," by M. G. Crosby, R.C.A. Communications, Inc., Riverhead, L. I., N. Y. (twenty minutes)
32. "Counter Circuits and Their Applications," by H. B. Deal, Radio Corporation of America, License Division Laboratory, New York, N. Y. (twenty minutes)

Detroit News

The Cranbrook Academy of Art will be visited by the women.

33. "Orthicon Portable Television Equipment," by M. A. Trainer, RCA Manufacturing Company, Inc., Camden, N. J. (twenty minutes)

10:30 A.M.–4:30 P.M.

Trip No. 4. Woman's sightseeing trip.

12:30 P-M.–4:30 P.M.

Trip No. 5. Luncheon at the Dodge or Plymouth plants and inspection of assembly lines.

BANQUET

The banquet will be held on Tuesday evening, June 24. Dress will be optional. Tickets will be $3.00 each. The Morris Liebmann Memorial Prize for 1941 will be presented during the banquet.

"Tricks with Speech and Song" will be presented by F. A. Firestone, Professor of Physics at the University of Michigan. A one-man quartet, speech without use of vocal cords, two voices for a single person, and a cello singing the words to a song are but some of the interesting demonstrations which he will present.

EXHIBITION

There will be no exhibition at this convention.

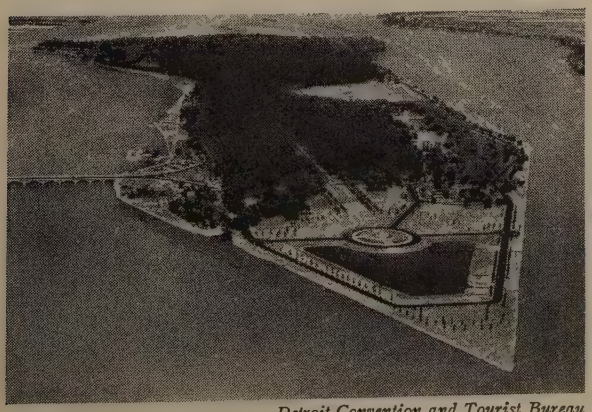
WOMEN'S PROGRAM

After registering, the women will meet in their headquarters at 10:30 A.M. for a brief discussion of their program. It should be noted that no program is scheduled for Tuesday morning.

Monday, June 23

TRIP NO. 1, SIGHTSEEING

At eleven o'clock, the busses will leave the Hotel Statler for a trip to Belle Isle, an island park of a thousand acres, which is located in the Detroit River at the mouth of Lake St. Claire. Stops will be made at the Conservatory and at the Nancy Brown Peace Carillon, where a brief concert will be given. From Belle Isle, the busses will go along Lake Shore Drive, Detroit's exclusive residential section, as far as the Grosse Pointe Yacht Club, and then to the Chrysler plant where the assembly of automobiles will be seen. Through the courtesy of the Chrysler Corporation, luncheon will be served and later, motion pictures will be shown in the Chrysler Auditorium.



Detroit Convention and Tourist Bureau

Air view of Bell Isle, a 1000-acre island park.

Entrance to Greenfield Village at Dearborn.

TRIP NO. 2, HARPER HOSPITAL

The women are invited to join the men on a trip, starting at 7:00 P.M. from the hotel, to Harper Hospital. This is one of Detroit's general hospitals and is a leader in the application of radio equipment and principles to therapy. A number of interesting demonstrations will be given by members of the staff.

Tuesday, June 24

TRIP NO. 3, EDISON INSTITUTE MUSEUM AND GREENFIELD VILLAGE

Busses will leave the hotel at 1:00 P.M. for the Edison Institute Museum and Greenfield Village.

The Edison Institute Museum houses an outstanding and large collection of colonial furniture, musical instruments, and household equipment. It is entered through a replica of Independence Hall.

At Greenfield Village will be found hundreds of interesting relics of American life. The village is composed of many structures originally built in colonial times. Spinning wheels, looms, instruments for rug making, quilting frames, costumes of all ages and for all occasions, cabinets, pottery, glass, and toys which were used in colonial America will be found in the various houses and shops of the Village.

Wednesday, June 25

TRIP NO. 4, SIGHTSEEING

Busses will leave the hotel at 10:30 A.M. for a trip out Woodward Avenue to the Bloomfield Hills district. A stop will be made at the Shrine of the Little Flower for a brief conducted tour. The next stop is at Will-O-Way Farm with its unique summer Straw Hat Theater in an air-conditioned apple storage barn. Treesa Way Merrill is planning a short program and Will-O-Way's famous apple cider will be served. Luncheon will be at Devon Gables, just a few rods from the theater. The bus trip then continues with the next stop at the Cranbrook

Detroit Convention and Tourist Bureau
Martha Mary Chapel in Greenfield Village.

Institute of Science for a conducted tour through its museum which is dedicated to the stimulation of interest in and interpretation of the physical world. Cranbrook offers unparalleled educational advantages to both boys and girls not only in the elementary grades but through college preparatory work. Cranbrook Academy will also be visited. The last place visited on the tour is Christ Church Cranbrook.

TRIPS

Trip No. 2, Harper Hospital

Busses will leave the hotel at 7:00 P.M. on Monday for Harper Hospital which is one of Detroit's leading general hospitals. For a number of years it has been active in the application of radio and electronic techniques to the problems of therapy and physical behavior. Equipment has been developed for the measurement of skin temperature, colorimetry, dielectric losses of the human body, and nerve and muscle chronaxie. A high-voltage X-Ray installation is used in the treatment of cancer. Some of the equipment will be demonstrated and its uses described.

Trip No. 3, Edison Institute Museum and Greenfield Village

On Tuesday, at 1:00 P.M., busses will leave the hotel on this trip.

The Edison Institute Museum presents through its display of equipment the history of the development of many industries. The electrical exhibition is unusually complete.

Of particular interest in Greenfield Village is the Menlo Park group of buildings. In the laboratory, which is part of this group, Edison developed the first incandescent light. Among the many other relics of colonial industry will be found the blacksmith shop, a grist and sawmill, and a wagon works and harness shop.

Trip No. 5, Plymouth or Dodge Plant

Busses will leave the hotel at 12:30 P.M. on Wednesday. The assembly line of either the Dodge or the Plymouth plants may be visited on this trip. It is expected that the Dodge plant will be turning out military cars for the United States Army and the Plymouth factory will be producing standard automobiles.

Luncheon will be served at both of the plants and tickets at seventy-five cents each should be obtained at the hotel before the start of the trip.

TECHNICAL PAPERS

Copies of technical papers will not be available either before or after the convention.

The number of papers has made it necessary to reduce to a minimum the time allotted for each. This time interval is noted for the individual papers in the preceding program. In addition, it has been found necessary to run parallel sessions on Monday afternoon and Tuesday morning. An attempt has been made to avoid any predictable conflicts in interests in the parallel sessions. It will be possible to pass freely from one meeting room to the other during the presentation of papers.

Summaries of the papers follow and are arranged alphabetically by the names of the authors. If there are two or more authors, the name of the first author determines the position of the summary. All papers are numbered in the order in which they are to be presented and these numbers are given with the summaries so the placement of each paper may be readily located.

SUMMARIES OF TECHNICAL PAPERS

5. ULTRA-HIGH FREQUENCY LOOP ANTENNAS FOR FREQUENCY-MODULATION BROADCASTING

Andrew Alford, A. G. Kandoian, and R. A. Hampshire
(International Telephone Development Company, Inc., New York, N.Y.)

Ultra-high-frequency loop antennas were first developed and are now widely used in radio aids to air

Detroit Convention and Tourist Bureau
Christ Church Cranbrook, twenty-five miles north of Detroit.

navigation. In this paper the use and advantages of these radiators for use as frequency-modulated broadcast antennas are discussed. It is shown that omnidirectional patterns of various amounts of gain may be obtained by stacking these loops. Design problems, efficiency, power handling capacity of single loop elements and arrays of frequency-modulation loops are considered.

3. A RADIO-FREQUENCY DEVICE FOR DETECTING THE PASSAGE OF A BULLET

C. I. Bradford
(Remington Arms Company, Inc., Bridgeport, Conn.)

An unusual application of radio engineering is described in which the reaction of a radio-frequency oscillator to the presence of a metallic projectile in a link-coupled coil is utilized in connection with projectile velocity measurements.

In order to obtain the accuracy of signal output required, the circuits are designed for good transient response. In addition, a unique method of differentiating, clipping, and amplifying is utilized in order to obtain a signal with steep wave front when the bullet is at the center of the coil regardless of the weight, shape, or velocity of the bullet.

Pictures are included of the equipment and coils used. High-speed pictures of bullets in the tripping position in the coils and oscillograms of circuit voltages verify that circuit operation is as predicted.

6. A TURNSTILE ANTENNA FOR ULTRA-HIGH-FREQUENCY BROADCASTING

George H. Brown and J. Epstein
(RCA Manufacturing Company, Inc. Camden, N. J.)

The purpose of this paper is to describe a new turnstile antenna design for ultra-high-frequency broadcasting. The original turnstile[1] was so constructed that the elements were fed by means of open wires twisting around the supporting mast. Adjustment of the phase relations and current magnitudes was accomplished by means of two properly matched transmission lines cut to the proper lengths and combined in a common terminal. The adjustment of these lines at the base of the antenna proper was rather critical and involved a certain amount of patient effort. With the advent of frequency modulation on the ultra-high frequencies, the turnstile antenna found many applications. It soon became apparent that many turnstile antennas would be placed on the tops of extremely tall supporting structures, where the matching and phasing adjustments become very difficult if not impossible. With these factors in mind, the development of a new type of turnstile was undertaken.

The most important feature of this new antenna is the fact that the antenna is completely pretuned in the fabrication of the individual elements so that no work of an engineering nature is necessary to put the antenna into operation.

The 90-degree phase relation is accomplished in the construction of the antenna elements themselves, so that no adjustments at the base are needed to obtain the circular pattern.

The antenna elements are so constructed that, while acting as insulated members for the radio-frequency signal, they are actually grounded to the pole to afford lightning protection. Because of the grounding feature, it is a simple matter to include sleet-melting units in each radiator.

Two concentric feed lines connect the several units, replacing the open wires formerly used. By means of standard concentric line, these two feed lines are fed in push-pull from a single concentric feed line which runs back to the transmitter. This feed line is terminated to eliminate standing waves.

The results of field tests of a six-layer turnstile of the improved design are discussed.

[1] George H. Brown, "A turnstile antenna for use at ultra-high frequencies," *Electronics*, April, 1936.

Cranbrook Academy of Art

Cranbrook Institute of Science which will be visited by the women.

Detroit Convention and Tourist Bureau

Shrine of the Little Flower, Royal Oak, Michigan.

7. FREQUENCY MODULATION FOR EMERGENCY COMMUNICATION

F. T. Budelman
(F. M. Link Company, New York, N. Y.)

The advantages of frequency modulation for emergency radio communication will be discussed. Field experience with a number of installations will be described, and design differences between emergency and broadcast transmitters will be pointed out.

11. CONTROL OF NIGHT ERROR IN AIRPLANE DIRECTION FINDING

H. Busignies
(International Telephone Development Company, Inc., New York, N. Y.)

This paper describes the development of a method enabling pilots to determine the accuracy of night bearings obtained by means of a radio compass in the wavelength range of 200 to 2000 meters (150 to 1500 kilocycles), and the effect on the indication when the plane passes through the combination of fields due to the reflection of waves from the E layer or from a mountain side. Consideration is given to the appearance of the night error on the ground and in altitude, taking into account the simultaneous presence of (1) the direct wave; (2) the sky wave; and (3) the sky wave reflected from the ground. It is demonstrated that the night error is smaller in the air than on the ground; also that there are regions in the atmosphere where the night error is very small.

Detroit News

Straw Hat summer theater at Will-O-Way Farm.

The dynamic aspect of the night error is then studied in the case of a plane moving through the above-mentioned system of waves. How the radio-compass indication changes regularly about a mean value, whether correct or not, according to the polarization of the sky wave, is next discussed. All cases of polarization are examined.

In conclusion, a number of rules are formulated relative to night direction finding on board airplanes above land or sea, supplemented by maps showing areas where direction finding is safe, unsafe, or dangerous. The maps show that the practical range of night direction finding is increased substantially by the correct interpretation of the radio-compass indications.

4. MOBILE TELEVISION EQUIPMENT

R. L. Campbell, R. E. Kessler, R. L. Rutherford, and K. U. Landsberg
(Allen B. DuMont Laboratories, Inc., Passaic, N. J.)

While portability is a necessary requirement for outside pickup equipment, several advantages result when portability is carried into the studio. To equip a studio of adequate size with fixed equipment for operation of several cameras involves considerable time and expenditure. However, with portable studio equipment, the entire equipment installation can be located to suit studio needs, as well as moved to different studios or outside locations.

The dolly-type equipment is described in some detail and systems for program control are discussed.

Some of the design features discussed are portable and flexible synchronizing equipment; electronic view finders; oscilloscope monitors; and other operating facilities.

31. OBSERVATIONS OF FREQUENCY-MODULATION PROPAGATION ON 26 MEGACYCLES

M. G. Crosby
(R.C.A. Communications, Inc., Riverhead, L. I., N. Y.)

A simplified analysis is given which shows that the distortion occurring with two-path transmission of frequency modulation consists of the introduction of a new modulation frequency which is frequency modulated and is not harmonically related to the fundamental modulation frequency. Results of observations at Riverhead, New York, on 26.3-megacycle frequency-modulated transmissions from W9XA at Kansas City are shown and discussed.

25. NEW SMALL ULTRA-HIGH-FREQUENCY RECEIVING TUBES

L. B. Curtis
(RCA Manufacturing Company, Inc., Harrison, N. J.)

New small receiving tubes combining the bulb and base structure of miniature tubes with electrode structures similar to unipotential-cathode-type acorn tubes are described. These new tubes are $\frac{3}{4}$ of an inch in diameter and $1\frac{7}{8}$ inches long, employ the convenient button-type base, and are simpler to manufacture than the acorn types. They are intended primarily for ultra-high-frequency applications where conventional tubes are inoperative. Performance results are discussed together with some of the factors affecting their operation at high frequencies.

32. COUNTER CIRCUITS AND THEIR APPLICATIONS

H. B. Deal
(Radio Corporation of America, License Division Laboratory, New York, N. Y.)

Simple and thoroughly satisfactory circuits for frequency multiplication are well known and widely practiced in the art. Completely satisfactory circuits for frequency division have been more difficult to achieve and are not in common use. Counter circuits are described in this paper as a means for obtaining division of frequencies below 1 megacycle, applicable to numerous purposes, and with advantageous performance in various respects.

21. THEORY OF RADIAL DIRECT-CURRENT SPACE-CHARGE FLOW BETWEEN CONCENTRIC CYLINDERS

W. G. Dow and A. B. Bronwell
(University of Michigan, Ann Arbor, Mich., and Northwestern Technological Institute, Evanston, Ill., respectively)

A general differential equation is set up governing the unidirectional direct-current and alternating-current flow of space charge wherever the lines of electric flux and space-charge flow are coincident and straight, thus including normally directed flow between parallel planes, radial flow between concentric cylinders, and radial flow between concentric spheres. It is shown that a simple form of these equations is obtained by the introduction of a parameter which measures transit time in terms of an electronic time unit of 2.38×10^{-8} second duration. Direct-current boundary conditions are classified as follows, for purposes of solution: Case I, involving a minimum in the potential distribution; Case II, in which the potential distribution curve rises from zero with a finite slope; and Case III, the complete space-charge condition. Numerically useful solutions for all of these three general types of direct-current flow between parallel planes and concentric cylinders are obtained, and presented in the form of parametric families of curves describing potential distribution and transit-time relations for useful ranges of currents, voltages, and geometries.

1. PHOTOGRAPHIC ANALYSIS OF TELEVISION IMAGES

D. G. Fink
(McGraw-Hill Publishing Company, New York, N. Y.)

The research reported in this paper was undertaken to develop a simple technique of studying the fine structure as well as the gross aspects of television image quality using photographic equipment. A standard television receiver and equipment for measuring the second-anode voltage, beam current, and peak-to-peak signal voltage were used in conjunction with a miniature camera having a lens of $f/1.5$ aperture, auxiliary lenses for close-up work, ground-glass focusing screen and magnifier, and a rigid adjustable copying stand. The speed ratings of several emulsions (Super XX, Plus-X, Panatomic-X Fine-grain Plenachrome, and Microfile) for light produced by the standard "P4" phosphor, were first determined in terms of beam current and second-anode voltage, as well as in terms of readings from a photoelectric exposure meter. These results, reported in tabular form, form a useful means of obtaining correct exposures under readily measurable conditions. The effect of the motion of a focal-plane shutter in photographing a scanned image was investigated. The equipment was found capable of recording a recognizable image solely from phosphorescent afterglow (exposure of 1/500 of a second during vertical blanking).

For studying fine structure, two auxiliary lenses of 1 and 2 diopters power were used to study line structure, interlace, pairing, effect of uniformity of field on vertical resolution, as well as the fine structure of horizontal resolution due to the desired signal and due to noise. Several conclusions based on this study are discussed.

A simple photographic standardization procedure was developed whereby the relative contrast and the average brightness of the fluorescent image could be determined from density measurements on the photographic negative. The effect of artificially enhancing the contrast during photographic processing is shown. The use of the equipment for documentary recording of program events is briefly illustrated.

13. A NEW AIR-COOLED 5-KILOWATT BROADCAST TRANSMITTER

F. W. Fischer
(Westinghouse Electric and Manufacturing Company, Baltimore, Md.)

A general description of the design and construction of the Westinghouse 5 HV transmitter will be given. Slides will be used to point out those features which will include among other details, the method of applying complete air cooling to the entire transmitter, the adaption of metal rectifiers in place of tube rectifiers, and the operation of a specially designed supervisory control system.

A discussion on the application of a new method of inverse feedback in the audio-frequency circuits will also be touched upon.

15. SHORT-WAVE SPREAD-BAND RECEIVER CIRCUITS

D. E. Foster and Garrard Mountjoy
(Radio Corporation of America, License Division Laboratory, New York, N. Y.)

Spread-band circuits most suitable for the antenna, radio-frequency, and oscillator stages of home and automobile receivers are described. Spread-band op-

eration requires departure from standard practice to achieve required performance characteristics, and these differences, which involve switching simplicity, number of components, and gain are treated. Expressions are given for gain, band width, and other factors vital to design work.

24. THE DESIGN AND DEVELOPMENT OF THREE NEW ULTRA-HIGH-FREQUENCY TRANSMITTING TUBES

C. E. Haller
(RCA Manufacturing Company, Inc., Harrison, N. J.)

A discussion and review are given of the service and design requirements of transmitting tubes intended for application in the ultra-high-frequency spectrum. These requirements fall in two classes: (1) those imposed by service conditions and (2) those imposed by the frequency at which the tube is operated. The fulfilling of these requirements has led to certain design and manufacturing problems, such as the reduction of grid emission, choice of anode material, choice of a suitable mechanical structure, etc.

A description of the novel features of construction and the operation of three new ultra-high-frequency transmitting tubes are also given. Two of these tubes are the RCA-815 and RCA-829 which are push-pull beam tetrodes while the third tube, the RCA-826, is a triode. Some precautions necessary for obtaining satisfactory operation with these tubes are given.

20. A MECHANICAL CALCULATOR FOR DIRECTIONAL ANTENNA PATTERNS

W. G. Hutton
(Station WGAR, Cleveland, Ohio)

The object of this paper is to describe a mechanical directional-antenna-pattern calculator to be used when all the factors are known that completely determine the pattern. This mechanical directional-antenna-pattern calculator is a machine which can be adjusted for any or all the factors that determine completely the directional pattern, and which on operation will automatically rotate the vectors that represent the relative antenna fields in such a way with respect to each other that the resultant vector or field ratios may be read directly.

The design problem for a directional-antenna pattern may involve the calculation of several trial patterns before the correct pattern is obtained. Thus a machine such as this paper describes will speed up and simplify design work for directional arrays.

The standard equation for field-intensity patterns is shown and an analysis is made to show that the results given by the machine is the solution to this equation.

The operating procedure is given for calculating the horizontal pattern with the machine described. The operating procedure is given for calculating the vertical-field pattern for any particular horizontal angle after the horizontal pattern has been figured by use of the machine.

The procedure is given for converting the vector values given by the machine to microvolts per meter both for the horizontal and vertical patterns.

30. MEASUREMENT, ANALYSIS, SYNTHESIS, AND EVALUATION OF THE SQUARE-WAVE RESPONSE OF TELEVISION APPARATUS

R. D. Kell, A. V. Bedford, G. L. Fredendall, and H. N. Kozanowski
(RCA Manufacturing Company, Camden, N. J.)

The sharpness of detail in a television picture is directly dependent upon the capability of the transmitter and receiver for the transmission of abrupt changes in picture half tone. Hence, the response of a television system to a precise square wave of relatively low frequency (100 kilocycles) is a criterion of picture detail. A portable high-frequency oscillograph has been developed by which a square-wave response may be viewed as a dotted wave and readily recorded as a series of readings. The dots, which are derived from the square-wave output of the circuit under test, are spaced at 1/30- (or 1/20-) microsecond intervals and provide a time calibration. No electrical connection is required between the oscillograph and the square-wave generator other than that established through the device under test.

Rules for the evaluation of the subjective sharpness of a picture from the square-wave response are given.

Rapid chart methods have been devised for (1) the analysis of a square-wave output into sine-wave amplitude and phase response and (2) the synthesis of a square-wave response from a given set of amplitude and phase characteristics.

The four aspects of the application of square waves to television—measurement, analysis, synthesis, and evaluation are presented as a basis for a unified and complete technique.

26. TRENDS IN RECEIVING-TUBE DESIGN

R. L. Kelly
(RCA Manufacturing Company, Inc., Harrison, N. J.)

The trends for receiving-tube design are toward improved high-frequency operation, smaller tube size, alternating-current—direct-current operation of small-size portable receivers, small-size push-pull receivers, and series heater operation. The new RCA 6SG7 and 12SG7 illustrate the effect of these trends. These tubes have high transconductance, low interelectrode capacitances, two cathode leads, and 0.3- and 0.15- ampere heaters, respectively. The performance of these tubes particularly at high frequencies is discussed.

The new RCA 45Z3 and 3Q4 miniature tubes exemplify the trend to small tube size. These two tubes

permit alternating-current—direct-current operation of small portable receivers employing the miniature tubes. Design features are covered.

Other tubes mentioned are the 6SF7, 12SF7, and 6SS7.

12. A METHOD OF CHANGING THE FREQUENCY OF A COMPLEX WAVE

E. L. Kent
(C. G. Conn, Ltd., Elkhart, Ind.)

This paper is a discussion and demonstration of two relatively simple means for changing the fundamental frequency of a complex wave retaining its original wave shape.

This method has its application in extending the range of the audio-frequency wave analyzer, electronic musical instruments, and possibly in the transmission of intelligence.

10. PHASE DISTORTION IN FREQUENCY-MODULATION SYSTEMS

N. I. Korman
(RCA Manufacturing Company, Camden, N. J.)

An appreciable amount of distortion may be generated by the curvature of the phase characteristic of the tuned circuits in a frequency-modulation system. The problem of calculating this distortion has been approached mathematically by several workers. The methods, however, have always been either beyond the abilities of anyone but a skilled mathematician to understand or else too tedious and laborious for practical use.

The difficulty of the mathematical approach led to the development of a physical approach which gives a comparatively simple method for the solution of the distortion problem. This simplified method gives a physical picture which is extremely helpful in developing and perfecting frequency-modulation equipment.

The simplified theory has been checked at a typical point against the more rigorous method of H. Roder and found to be in excellent agreement with it.

14. OPTIMUM CURRENT DISTRIBUTIONS ON VERTICAL ANTENNAS

Lincoln La Paz and G. A. Miller
(Ohio State University, Columbus, Ohio, and the National Research Council, Ottawa, Ont., Canada, respectively.)

The theoretical optimum current distribution on a vertical antenna of given length is defined as that current distribution giving the maximum possible field strength on the horizon for a given power output.

It is pointed out that such a distribution is well worth determining, even though it may be impossible to realize it in practice, for the two following reasons:

1. The apparent antenna performance obtained with the theoretical optimum distribution is as good or better than that obtained with any practical distribution, and thus serves to bound the improvement in antenna performance which may be expected in the future as a result of changes in current distribution.

2. The theoretical optimum-current distribution gives an indication of the general class of distribution most likely to yield worth-while results in a search for practical optimum distributions.

It is shown that the theoretical optimum current distribution is of necessity a constant-phase distribution; i.e., the current at every point on the antenna is either in phase or 180 degrees out of phase with the current at every other point on the antenna. Thus it is necessary to consider only constant-phase-current distributions.

It is further pointed out that the problem of determining the theoretical optimum current distribution is mathematically equivalent to the problem of determining that current distribution giving the minimum possible power output for a fixed field strength on the horizon. An admissible current distribution is defined as any current distribution $f(x)$, continuous from O to L, and giving the required field strength on the horizon; or in other words satisfying the condition

$$\int_0^L f(x)dx = K$$

where L is the electrical length of the antenna and K is a constant.

The problem of determining the theoretical optimum current distribution on a vertical antenna is set up in the form of a problem in the calculus of variations, and the following necessary and sufficient condition for an exact solution is derived:

If the proposed problem actually has an exact solution in the class of admissible functions $\{f\}$, then a necessary and sufficient condition that an admissible function $f(x)$ be a solution is that it also be a solution of the integral equation

$$J(y,f) = \int_0^L f(x)\left[\frac{\sin(x-y)-(x-y)\cos(x-y)}{(x-y)^3} + \frac{\sin(x+y)-(x+y)\cos(x+y)}{(x+y)^3}\right]dx$$

$$= \text{constant for } 0 \leqq y \leqq L.$$

It is shown that all functions $f(x)$ which satisfy this condition approximately, are certainly approximate solutions to the problem, whether the problem actually possesses exact solutions or not.

Approximate theoretical optimum current distributions are derived from this condition for antennas of various heights from $\frac{1}{8}$ of a wavelength up to a full wavelength. Finally a curve of maximum possible field strength on the horizon for a fixed power output is given as a function of antenna height, thus bounding the problem.

8. A NEW FREQUENCY-MODULATION TRANSMITTER

N. C. Olmstead and A. A. Skene
(Bell Telephone Laboratories, Inc., New York, N. Y.)

A new frequency-modulation transmitter is described which uses a novel amplifier circuit permitting an unusually simple mechanical design and an economical vacuum-tube complement.

The choice and design of circuit components, governed by both mechanical and electrical considerations, are discussed in detail.

27. THE EFFECTS OF CONTACT POTENTIALS ON THE CHARACTERISTICS OF VACUUM TUBES

G. D. O'Neill
(Hygrade Sylvania Corporation, Salem, Mass.)

Bridge measurements of the dynamic and static characteristics of vacuum tubes furnish sufficient information for computing a voltage E' which comprises the voltage equivalent of the effects of initial electron velocities and contact potential difference.

The effects of aging and use on contact potential and E', and the resulting effects on the static and dynamic characteristics of tubes, are discussed.

Knowledge of the values of E' in samples is of considerable importance in the design of tubes and standardization of their characteristics. Examples of the methods used in the solution of practical problems are given.

18. TRANSMISSION OF AN ELECTROMAGNETIC WAVE BY A ROW OF EQUIDISTANT SIMILAR PLATES

Hillel Poritsky
(General Electric Company, Schenectady, N. Y.)

In this paper the transmission characteristics to plane electromagnetic waves of a row of equidistant similar plates are considered. First a general solution of the electromagnetic equations in a gap and an adjacent plate satisfying the proper continuity conditions at the interface is obtained. This solution contains two arbitrary constants. Two characteristic solutions are next found which have the property of being multiplied by a constant λ when one increases the linear coordinate normal to the plate surfaces by a period equal to the joint width of a plate and a gap. The equation for determining λ is an algebraic equation of the second degree whose roots are reciprocals of each other. When these roots are complex they are of unit amplitude; when they are real, one is, in general, numerically greater than 1 and the other numerically less than 1. From the above it follows that the row of plates acts like a filter in electric-circuit theory, the case of real values of λ corresponding to an attenuation band, the case of complex values to a transmission band.

An analogy is pointed out between this problem and that of circuits with variable parameters, the case at hand corresponding to a circuit whose capacitance is changed periodically from one constant value to another one. As is known, a circuit of this kind is sometimes stable and sometimes unstable; the former case corresponds to a transmission band with a row of plates, the latter to an attenuation band.

The same situation applies to general media with a periodically varying index of refraction.

2. INDUSTRIAL ELECTRONIC APPLICATIONS

R. A. Powers
(Electronic Control Corporation, Detroit, Mich.)

Many industrial processes may be sped up or made more accurate by the utilization of photoelectric devices as control elements. High accuracy of control may be obtained. Additional uses include the counting of small objects when being packaged, the rejection of off-color products, and smoke detection.

Small Thyratrons have been used for measuring, grading, and sorting. They have been substituted for mechanical relays for sequential switching operations to provide more rapid operation.

Safety applications of these devices will be described.

19. THE FIELD-THEORY APPROACH TO NONUNIFORM TRANSMISSION LINES

Simon Ramo
(General Electric Company, Schenectady, N. Y.)

Nonuniform-line analysis has ordinarily been based upon the notion of a distributed impedance and admittance just as in uniform-line studies, the equations including, however, the dependence of these constants upon distance along the line. To examine the validity of this approach to the uniform-line problem, rigorous analyses have been made in the past by use of Maxwell's field equations. These analyses showed that for perfect conductors the electric- and magnetic-field components of the principal wave in the direction of propagation vanish everywhere in space and only a transverse electric field between conductors and a transverse magnetic field linking the conductors exist. Furthermore, the effects of sinusoidal changes with time and distance co-operate in such a way as to yield the distributions of these fields identical with those of the static case. Now in a nonuniform line there is even less reason for immediate satisfaction with the distributed-constant point of view, especially at ultra-high frequencies, where the distance between lines is not likely to be infinitesimal compared to wavelength. If the distributed-constant concept is used, it would appear that mutual inductances and mutual capacitances should be considered.

The present paper uses the rigorous field equations to examine transmission properties along general, non-

uniform lines having symmetry about some axis. It is shown that the electric field of the principal wave will not be completely transverse in every case, and the charges on opposite faces of the conductors, as well as the current flow along them, will not be equal and opposite. This indicates the general need for mutual inductances and capacitances in analysis by conventional methods. It is found, however, that for some configurations the electric field in the direction of the conductors vanishes everywhere. When this occurs, the currents in the two conductors are always equal and opposite, but "equal and opposite" must refer to points on the same orthogonal surface perpendicular to the conductors rather than at the same point along the line. This condition says that certain nonuniform lines may be accurately analyzed without considering distributed mutual impedances. The presence of mutual effects is seen not to be a requirement in yielding transmission characteristics which differ markedly from those of uniform lines. Finally, examples are discussed showing the applicability of field theory to studies of propagation between co-axial cones, spheres, disks, ellipsoids, and hyperboloids.

16. FACTORY ALIGNMENT EQUIPMENT FOR FREQUENCY-MODULATION RECEIVERS

H. E. Rice
(Stromberg-Carlson Telephone Manufacturing Company, Rochester, N. Y.)

A description of some of the problems arising in alignment of frequency-modulation receivers is given and a method of meeting these problems with a minimum of difficulty and complexity is shown.

29. THE RELATIVE SENSITIVITIES OF TELEVISION PICKUP TUBES, PHOTOGRAPHIC FILM, AND THE HUMAN EYE

Albert Rose
(RCA Manufacturing Company, Inc., Harrison, N. J.)

The question of the relative sensitivities of television pickup tubes and photographic film has frequently been raised in discussions of the performance of television pickup tubes. In these discussions, sensitivities have usually been compared in terms of the lowest scene illumination at which the given device can still "see." While this is a useful criterion for operation of the device, it is not a complete definition of sensitivity. The "operating sensitivity" as defined above depends upon the lens system, exposure time, and amount of detail demanded in the reproduced image. Although the lens system and exposure time have usually been taken into account, the amount of detail in the reproduced image has not been given the same attention.

For the purpose of this discussion it has been found useful to define a more intrinsic measure of sensitivity than that contained in the definition of "operating sensitivity." This measure is the reciprocal of the threshold number of lumens required per picture element of the reproduced image and will be denoted by the term "true sensitivity." The "operating sensitivity" may be easily derived from the "true sensitivity" when the f number of the lens, the exposure time, and the image detail are specified. The "true sensitivities" of pickup tubes have been calculated from the quantum efficiency of the photoelectric process and the magnitude of fluctuation phenomena either in the preamplifier or in the photoelectric process. The "true sensitivities" of photographic film and of the human eye have been estimated on the basis of published data for the number of quanta necessary to render a grain developable and the number necessary to excite a retinal element. If one takes the picture element of film to be of the order of a grain, the results of this analysis indicate that eye and film have about the same "true sensitivity" and both are from 1/10 to 1/100 as sensitive as a perfect device. A perfect device is defined as one which has 100 per cent quantum efficiency, makes full use of the exposure time, and is limited only by the fluctuations in the arrival of light quanta. Both eye and film have approximately 10,000 times the "true sensitivity" of some recent television pickup tubes.

To compare "operating sensitivities," the same exposure time and equivalent lens systems are taken for the three devices. A television pickup tube which has a photoelectric response of 10 microamperes per lumen and makes full use of the storage principle can record scenes with no more illumination than that required by some of the "faster" photographic films. This is understandable since the film has used much of its relatively high "true sensitivity" to provide correspondingly more detail than is found in present television pictures. The human eye has a range of "operating sensitivities" extending from that of film to a value several thousand times higher. This range of "operating sensitivities" depends upon the ability of the eye to coarsen the detail of its perceived image as the scene brightness is lowered.

28. A METHOD FOR CALCULATING THE PERFORMANCE OF SELF-BIASED PLATE-MODULATED AMPLIFIERS

R. I. Sarbacher
(Illinois Institute of Technology, Chicago, Ill.)

A method has been developed by means of which it is possible to calculate the performance of a plate-modulated amplifier with a resistance bias. This method permits the accurate determination of the path followed by the quiescent point during the modulation cycle when the bias for the amplifier is obtained either from grid current alone or from grid and plate current together.

Dynamic characteristics showing the performance of a triode operating as a plate-modulated amplifier have been obtained for generator bias, resistance bias, and a combination of both. The improvement in linearity of the amplifier and in the reduction in driving power required when resistance bias in the grid circuit gradually replaces generator bias is clearly shown on these dynamic characteristics. A large reduction in peak driving power is also plainly evident when grid-resistance bias is employed. The gains and sacrifices made in the transition from one type of grid polarization to the other are brought out. An experimental confirmation of the calculated characteristics shows an agreement everywhere within 3 per cent.

A calculating device has been developed which aids materially in the calculation of the operating characteristics of amplifiers operating in class B or C. This device, employed in the calculation of the material discussed above, greatly simplified the procedure and speeded the progress of the work.

23. DEIONIZATION CONSIDERATIONS IN A HARMONIC PRODUCER EMPLOYING A GAS-TUBE SWITCH

W. G. Shepherd
(Bell Telephone Laboratories, Inc., New York, N. Y.)

A description is given of an experimental investigation of the properties of a Thyratron operating as a high-frequency switch in a circuit which permits the generation of a wide band of harmonics. It has been found that there is an operating frequency below which no difficulties in deionization occur and above which stable operation requires that the grid potential fulfill certain conditions dependent upon the frequency, wave form of the grid voltage, and circuit constants. It has been found possible to operate certain standard Thyratrons at switching frequencies as high as several hundred kilocycles per second. For these higher frequencies the deionization of the tubes is incomplete but normal switching behavior is obtained.

22. SOME SIMPLIFIED METHODS OF DETERMINING THE OPTICAL CHARACTERISTICS OF ELECTRON LENSES

Karl Spangenberg and L. M. Field
(Stanford University, Stanford University, Calif.)

Some new methods of calculating lens characteristics are proposed which are relatively simpler and more accurate than previously proposed methods. The first method is an extension of the method of joined circular segments of Salinger applied to paraxial rays in fields with a rotational symmetry. This requires as information only the axial potential and derivatives thereof. The method is the computational equivalent of the original graphical method. A second method makes use of the action function which is approximated from the potential function. Electron paths are taken as normal to the lines of constant action. A third method replaces the convergent and divergent parts of the usual lens with equivalent thin lenses and then calculates the focal lengths by means of combination formulas applied to the two thin lenses. All calculating methods are, however, sufficiently long in application and indeterminate in accuracy that experimental methods of finding lens characteristics are preferred.

A new experimental method makes use of a demountable vacuum tube. Lens characteristics are determined from angular magnifications measured from the shadows cast by object screens illuminated by a point source of electrons. No moving screens are required nor is it necessary to generate rays parallel to the axis. By observing magnifications for all voltage ratios for two positions of the object screen enough data are available to determine the four cardinal focal distances for all voltage ratios. The results are considered more accurate and cover a greater range of voltage ratios than those reported by previous investigators. A graphical method has been developed for determining the spherical-aberration characteristics of the lens from the curvature of the object screen images observed on the flourescent screen.

Results are presented in a new and simplified form in addition to the standard form showing the variation of focal distances with voltage ratio. The new form gives associated object and image distances and corresponding magnification for any voltage ratio. The relation between the four useful variables is thus given on one chart for each lens. In effect this new form is a graphical presentation of the complete solution of the lens equation which shows clearly the relation between the four associated variables and gives quantitative results which can be applied directly. The new form reveals a number of properties of lens strength and magnification which have not been previously recognized. An approximate universal magnification formula is stated.

A design procedure for cathode-ray tubes is given. With the help of the new object-image-distance curves the relation between all quantities is given directly and it is no longer necessary to resort to a set of trail-and-error calculations to arrive at a satisfactory combination of variables.

9. A FREQUENCY-MODULATION STATION MONITOR

Harry R. Summerhayes, Jr.
(General Electric Company, Schenectady, N. Y.)

This is a nonmathematical paper which describes the development, theory, and characteristics of a frequency-modulation station monitor developed in the General Engineering Laboratory of the General Electric Company. The monitor measures center frequency

with and without modulation and the percentage of frequency modulation as a per cent of 75-kilocycle deviation. In addition, it provides a flasher and alarm circuit for overmodulation adjustable from 50 to 120 per cent and an audio-frequency-fidelity monitoring circuit. The paper begins with a discussion of the requirements for frequency-modulation monitors and compares the problems involved with those involved in standard broadcast monitors. Several schemes which were examined for use as a frequency-modulation monitor are briefly described, including counter and frequency-division systems. The principles and design of the final monitor are discussed. In these monitors, precision crystal oscillators are employed to provide for both accurate calibration and the intermediate frequency.

The last part of the paper is devoted to a discussion of the salient development problems encountered, to a detailed description of the circuit, and to a discussion of the over-all specifications of the commercial sample.

33. ORTHICON PORTABLE TELEVISION EQUIPMENT

M. A. Trainer
(RCA Manufacturing Company, Inc., Camden, N. J.)

Recently developed Orthicon portable television equipment is described. Because of the greater light sensitivity of the Orthicon, it is expected that the equipment will fill a need for lightweight equipment to be used under adverse light conditions. Among the novel features of the design are the use of a forced-air-cooled transformer in the regulated power-supply unit, an all-electronic synchronizing generator, gamma control, and keyed diodes for black-level setting.

The units required for a one-camera setup are camera and tripod, camera-control unit, power-supply unit, pulse unit, and shaping unit. The total weight, exclusive of camera cable, is 370 pounds and the power required, 1250 watts.

17. THE FULL-WAVE VOLTAGE-DOUBLING RECTIFIER CIRCUIT

D. L. Waidelich
(University of Missouri, Columbia, Mo.)

An analysis of the full-wave voltage-doubling rectifier circuit is made in this paper with the main assumption that the tube drop is zero when conducting. Both the output and the input performance characteristics of the circuit are presented, and several of the characteristics are compared with experimental results. The analysis shows that polarized electrolytic condensers may be used if the circuit is not loaded too heavily. The currents to be expected on short circuit are also discussed.

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Spurious Signals

The article by Eastman and Horle on "Spurious Signals" in the October, 1940, PROCEEDINGS, reminds me of an experience in the days of crystal sets.

About fifteen years ago, I had two crystal sets in my house, located in separate rooms and connected to separate antennas. I was listening on one set one evening, when during a silent interval I heard a "kuh" in the receiver and almost simultaneously heard a cough through the open door of the room where the other set was located.

I called to the other room and had the noise repeated and again heard it in the receiver. Further tests revealed that the action was "two-way" and that we could actually carry on a two-way conversation between the two sets by talking into the receivers.

Further investigation revealed that both sets were tuned to KLX which was a 1-kilowatt station about 4 miles distant. Also that if the sets were tuned to different stations, no signals could be interchanged.

The possibility of cross talk from adjacent wiring, defective grounds, or capacitive coupling between the antennas (each about 100 feet long and 50 feet apart) was eliminated by test.

It was found also that this effect could be produced by tuning both sets to any near-by station, the magnitude of the effect being proportional to the strength of the station signal.

Finally, a one-way conversation was conducted over a distance of about 600 feet by tuning the transmitting and receiving sets to KLX and then substituting a carbon transmitter for the receiver of the transmitting set.

These conversations were plainly audible and intelligible during silent intervals of the broadcast station but were masked during the program periods.

Evidently the two antenna systems were oscillating under the impulse of the near-by broadcast station and the rebroadcast was "stealing a ride" on the carrier wave. The modulating effect of the voice currents on the crystal detector was presumably similar to that of the copper-oxide modulators in modern systems.

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Books

Geophysical Exploration, by C. A. Heiland

Published by Prentice-Hall, Inc., New York, N. Y., 1940. 963 pages. Index, 48 pages. 535 figures. 6¼×9¼ inches. Price, $10.00.

This book covers the whole field of geophysical exploration which the author defines as, "prospecting for mineral deposits and geological structures by surface measurements of physical quantities." Thus, the past and future development of geophysical exploration is intimately associated, on the one hand, with geology and, on the other, with physics.

Both students and practitioners have not had available desirable technical literature in the form of books dealing exclusively with geophysics and geophysical exploration. It is rather surprising that this need has not been met previously, in view of the fact that petroleum geophysics is some 15 or 20 years old, and has a very great economic value.

Professor Heiland's book is one of three recent publications dealing with this spe-

cial subject. He is to be commended most highly for devoting so much time to the preparation of a book which includes so much material. As head of the department of geophysics at the Colorado School of Mines, and as a director of geophysical exploration in so many parts of the world, he is well qualified to develop both the theoretical and practical aspects of geophysical exploration. Both of these two aspects are well developed in his book of one thousand pages.

The book is divided into two parts. One gives, in a concise manner, an introduction of some 65 pages dealing with the fields of applied geophysics. Part two covers six divisions of exploration involving the following methods:

1. Gravitational
2. Magnetic
3. Seismographical
4. Electrical
5. Oil-well testing
6. Miscellaneous geophysical methods.

Most geophysical instruments which are used in the various methods are described and copiously illustrated with photographs. From the view of research and development on geophysical instruments, the book leaves much to be desired. In geophysical exploration, so much depends upon the instruments, and in turn, they depend so much upon a comprehensive knowledge of the fundamental scientific factors involved. These subjects are not treated in as complete a manner as would seem desirable. However, such a discussion was not intended on the part of the author inasmuch as he states that the purpose is to cover geophysical exploration and the interpretative techniques of the physical data. These are presented in much detail.

This book is perhaps prepared for the petroleum geophysicists rather than for geophysicists in general, and from the viewpoint of the former, the gravity meter and seismograph are the two most potent instruments.

The author is one of the first to have devoted considerable space to the modern gravity meter which, in the past few years, has rapidly surpassed the torsion balance in the art of gravitation prospecting. Although the general instrumental features of several gravity meters of both the astatized and unastatized types are presented, the important subjects of temperature control and of highly elastic materials are not discussed. The reduction of field data for purposes of interpretation is ably presented. Treatment of the torsion balance as a gravitational instrument is quite complete and presented with considerable clarity.

The modern seismograph used in petroleum exploration work has been adapted to the requirements of both refraction and reflection shooting. Both of these are of much importance in exploration work. The instruments used in these methods consists of geophones, vacuum tubes, amplifiers, volume and frequency controls, transmission and filter networks, wire and radio communication, etc. These various subjects are presented briefly in the book, but the geophysicists must refer to the many books and periodicals contributed by the radio and communication engineers.

Elastic wave propagation in the earth is presented from the classical viewpoint. But the propagation of transient seismic waves of the various types, due to detonation of dynamite in the ground, in the heterogeneous subsurface geological formations requires a very substantial extension over the classical wave theory. Such an extension of wave theory is, no doubt, beyond the scope of the book, although the extension is of much value to the practical accomplishment of the seismograph methods.

There is as much space given to the magnetic and electric methods as there is to the gravity and seismic. But, neither of the two former methods have, as yet, proved of economic value comparable to the two latter ones.

The author would have difficulty in securing agreement to his view that chemical analyses applied to surface and subsurface geological formations could be included as a branch of geophysics. We may see geochemistry and geophysical chemistry become a major development in their own rights.

Radioactive methods of exploration which have lately gained in importance are historically reviewed, but recent use of the sensitive ionization chamber is not included.

The reviewer could not help but compare Professor Heiland's *Geophysical Exploration* with Professor Morecroft's *Principle of Radio Communication*. What the latter is to radio, so the former is to geophysics.

J. P. Minton
Magnolia Petroleum Co.
Dallas, Texas

Electron Inertia Effects, by F. B. Llewellyn

One of the Cambridge Physical Tracts published by the Cambridge University Press, 60 Fifth Ave., New York, N. Y. Price $1.75. 102 pages, 13 figures.

This monograph recommends itself to all concerned with the behavior of the lethargic electron at high frequencies. Twelve years ago W. E. Benham's brilliant analytic methods showed us quantitatively, for the first time, some of the extraordinary turns we must expect in vacuum-tube performance when the electron transit time is comparable with the reciprocal of the operating frequency. Today the design and application of vacuum tubes for high-frequency purposes rests squarely upon a solid body of theory, to which Llewellyn has been a prominent contributor.

Here in eight chapters, he has integrated the content of several papers to furnish in one convenient volume the gist of the new theory. An introductory chapter exhibits the need for an analysis distinct from that appropriate to classical effects of electron inertia, such as polarization and optical properties of materials. Following it come three chapters developing the general theory of electron motion between parallel planes. This material has been extracted, almost entirely, from the author's original papers, with few changes except for beneficial alterations in symbols and re-ordering for maximum coherence. The theory is then brought to bear upon the small-signal performance of diodes and negative-grid triodes, with attention particularly directed towards the devising of equivalent circuits, the production of negative-resistance oscillations, the nature of active grid loss, and the modification of electrostatic capacitance by the presence of space charge. There follows a short application to small-signal rectification, and a description of the necessary correction to vacuum-tube-voltmeter readings at the threshold of transit-time entanglement. The final chapter indicates the extent to which the theory must be stretched to fit cylindrical structures, and touches upon magnetrons and Barkhausen-Kurz oscillators only long enough to show in brief how their performance depends upon transit time.

In a general preface the editors state that "each author is encouraged to adopt an individualistic outlook and to write the tract from his own point of view without necessarily making it 'complete' by the inclusion of references to all other workers or to all allied subjects; it is hoped that the tracts may present such surveys of subjects as the authors might give in a short course of specialized lectures. By this means readers will be provided with accounts of those subjects which are advancing so rapidly that a full-length book would be out of place. From time to time it is hoped to issue new editions of tracts dealing with subjects in which the advance is most rapid." Within these confines the volume has been ably prepared. It would be unkind indeed to complain that little material of an experimental nature has been included, and no mention made of the infant theory of electron beams in application to such new arrivals as "velocity-modulation" devices, "inductive-output" tubes, and the general theory of excitation of resonant cavities. One may hope however, that the editors will not overlook these equally fascinating aspects of the subject.

To the novice determined to absorb the most with the least digestive disturbance, and to the specialist who prefers his reference materials in compact form this tract is earnestly commended.

Dwight O. North
RCA Manufacturing Company, Inc.
Harrison, N. J.

Electrodynamics, by Leigh Page and Norman Ilsley Adams, Jr.

Published by D. Van Nostrand Co., 250 Fourth Ave., New York, N. Y. 506 pages. 115 figures. Price $6.50.

This is a book on the theoretical aspects of electrodynamics for the mathematical physicist or the advanced student of physics. Electrostatics and magnetostatics are not considered, except where they appear as incidental to the treatment of the electromagnetic field.

No detailed description of the material included in such a distinctly mathematical treatise is possible here, but a list of the main sections of the book may be helpful.

The book begins with a thorough and extended treatment of three-dimensional vector analysis in which are introduced

the fundamental mathematical theorems and operators which are necessary tools in the development of the sections which follow. This section is followed by one on the principle of relativity. The equations of the electromagentic field are deduced on the basis of the emission theory. A section on the elementary charge and the force equation leads up to a consideration of the phenomena in material media. Radiation and radiating systems are treated at length and a section is devoted to the electromagnetic theory of light. The book ends with chapters on four-dimensional analysis and on general dynamical methods.

The book is an example of beautiful printing of mathematical material. The well-known work of the authors in the fields covered in the book ensures that the treatment is authoritative and up to date.

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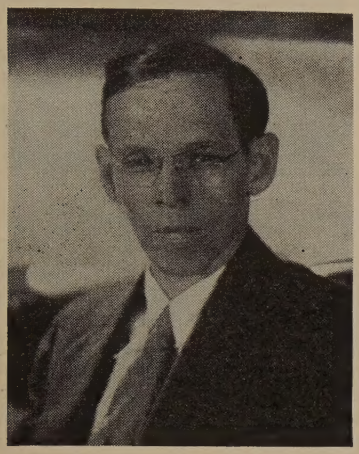
Contributors

Hugh A. Brown (A'16–M'29) was born at Cedar Springs, Pennsylvania, on June 19, 1889. He received the B.S., M.S., and E.E. degrees from the University of Illinois, and began teaching in 1915. Since



Hugh A. Brown

1926 he has devoted considerable time to research on methods of detecting and measuring ionization discharge in gaseous-voids insulation, and later, on corona discharge on the surface of insulation, carrying out the development of the discharge-detection bridge for use at both power and radio frequencies. Dr. Brown is professor of electrical engineering at the University of Illinois. He is a member of Sigma Xi and the American Institute of Electrical Engineers.

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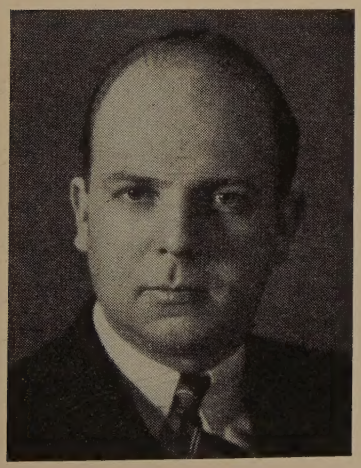

W. L. Everitt

William L. Everitt (A'25–M'29–F'38) received the E.E. degree from Cornell University in 1922, the M.S. degree from the University of Michigan in 1926, and the Ph.D. degree from Ohio State University in 1933. He has taught electrical engineering at Cornell, Michigan, and Ohio State and has been in charge of the instruction in communications at the last institution since 1926. Dr. Everitt initiated and has directed the annual Broadcast Engineering Conference at Ohio State. He is a Fellow of the American Institute of Electrical Engineers and a member of the National Defense Research Committee.

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James Hillier

James Hillier was born in Brantford, Ontario, on August 22, 1915. He received the degree of B.A. in physics in 1937, and M.A. in physics in 1938 from the University of Toronto, Canada. From 1939 to 1940, Mr. Hillier was a research assistant and demonstrator at the Banting Institute of the University of Toronto. Since 1940 he has been a research engineer at the RCA Manufacturing Company, Camden, N.J.

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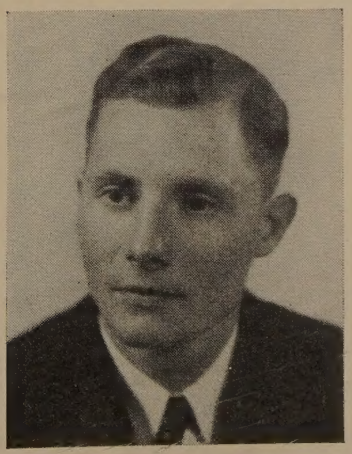
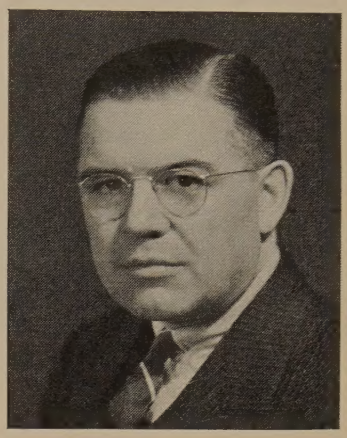
E. C. Jordan (S'36–A'40) was born on December 31, 1910, at Edmonton, Canada. He received the B.Sc. degree in 1934



E. C. Jordan

and the M.Sc. degree in 1936 from the University of Alberta, and the Ph.D. degree in 1940 from Ohio State University. From 1928 to 1935 Dr. Jordan was control operator of CKUA, and during the



John H. Little

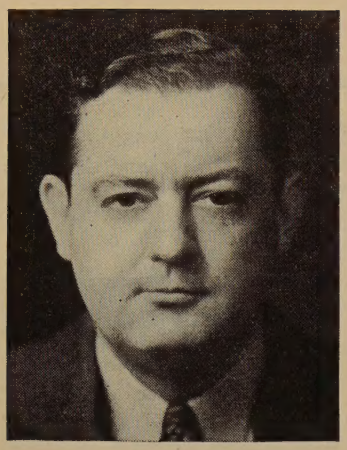
summers of 1931 and 1932, he was with the Alberta Government Telephones. During 1936 and 1937 he was a junior electrical engineer with the International Nickel Company; from 1938 to 1939, a University Scholar; and in 1940, a Battelle Research Associate at Ohio State University. At the present time, Dr. Jordan is an instructor in electrical engineering at Worcester Polytechnic Institute. He is a member of Tau Beta Pi and Sigma Xi.

❖

John H. Little was born at Lockport, New York. He received the B.S. degree in

F. X. Rettenmeyer

electrochemical engineering from Massachusetts Institute of Technology in 1923. Mr. Little was a research chemical engineer with Johns-Manville, Inc., from 1923 to 1925; in the General Motors research laboratory from 1925 to 1927; assistant head



A. W. Vance

of the lighting section of General Motors research laboratory from 1927 to 1933; and assistant electrical engineer, Chevrolet Division, from 1933 to 1935. Since 1935, Mr. Little has been electrical engineer and accessory engineer in the Chevrolet Division of General Motors.

❖

F. X. Rettenmeyer (A'26–M'29) was born in Oklahoma. He received the E.E. degree from the University of Colorado in 1922 and in the same year, joined the Western Electric Company. In 1930, following a period with F.A.D. Andrea, Inc., Mr. Rettenmeyer entered the Bell Telephone Laboratories where he remained until 1935 when he was placed in charge of the receiver department of the engineering division of the Radio Corporation of America.

❖

Arthur William Vance was born on July 15, 1904. He was graduated from Kansas State College in 1928 with the degree of B.S. in electrical engineering. From 1929 to 1930 he was a television and facsimile research engineer at the Westinghouse Electrical and Manufacturing Company. Since 1930, Mr. Vance has been employed as engineer in charge of circuit research in the Electronics Research Division of the RCA Manufacturing Company.

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Chao-Chen Wang was born on October 21, 1914, at Changchow, Kiangsu, China. He received the S.B. degree in electrical engineering from Chiao-Tung University in Shanghai in 1936. Dr. Wang held the exemption scholarships several times for high-average record in the college and he was awarded a membership in the Phi Tau Phi Society of the college on graduation.

Chao-Chen Wang

During 1936, he was a junior engineer of the Artillery Research Division of the War Ministry of the Chinese government. In 1937, Mr. Wang received the Tsing-Hua University Fellowship for advance study in the United States. He entered the graduate school of engineering of Harvard University where he received the S.M. and Sc.D. degrees in communication engineering in 1938 and 1940, respectively. He is now visiting the RCA Manufacturing Company at Harrison, New Jersey.